

Discrete cosine transform

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The **discrete cosine transform** (DCT) is a Fourier-related transform similar to the discrete Fourier transform (DFT), but using only real numbers. It is equivalent to a DFT of roughly twice the length, operating on real data with even symmetry (since the Fourier transform of a real and even function is real and even), where in some variants the input and/or output data are shifted by half a sample. (There are eight standard variants, of which four are common.)

The most common variant of discrete cosine transform is the type-II DCT, which is often called simply "the DCT"; its inverse, the type-III DCT, is correspondingly often called simply "the inverse DCT" or "the IDCT".

Two related transforms are the discrete sine transform (DST), which is equivalent to a DFT of real and *odd* functions, and the modified discrete cosine transform (MDCT), which is based on a DCT of *overlapping* data.

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Applications

The DCT, and in particular the DCT-II, is often used in signal and image processing, especially for lossy data compression, because it has a strong "energy compaction" property: most of the signal information tends to be concentrated in a few low-frequency components of the DCT, approaching the Karhunen-Loève transform (which is optimal in the decorrelation sense) for signals based on certain limits of Markov processes.

For example, the DCT is used in JPEG image compression, MJPEG video compression, and MPEG video compression. There, the two-dimensional DCT-II of 8x8 blocks is computed and the results are quantized and entropy coded. In this case, *n* is typically 8 and the DCT-II formula is applied to each row and column of the block. The result is an 8x8 transform coefficient array in which the (0,0) element is the DC (zero-frequency) component and entries with increasing vertical and horizontal index values represent higher vertical and horizontal spatial frequencies.

A related transform, the *modified* discrete cosine transform, or MDCT, is used in AAC, Vorbis, and MP3 audio compression.

DCTs are also widely employed in solving partial differential equations by spectral methods, where the different variants of the DCT correspond to slightly different even/odd boundary conditions at the two ends of the array

Formal definition

Formally, the discrete cosine transform is a linear, invertible function $F : \mathbf{R}^n \rightarrow \mathbf{R}^n$ (where \mathbf{R} denotes the set

of real numbers), or equivalently an $n \times n$ square matrix. There are several variants of the DCT with slightly modified definitions. The n real numbers x_0, \dots, x_{n-1} are transformed into the n real numbers f_0, \dots, f_{n-1} according to one of the formulas:

DCT-I

$$f_j = \frac{1}{2}(x_0 + (-1)^j x_{n-1}) + \sum_{k=1}^{n-2} x_k \cos \left[\frac{\pi}{n-1} j k \right]$$

Some authors further multiply the x_0 and x_{n-1} terms by $\sqrt{2}$, and correspondingly multiply the f_0 and f_{n-1} terms by $1/\sqrt{2}$. This makes the DCT-I matrix orthogonal (up to a scale factor), but breaks the direct correspondence with a real-even DFT.

A DCT-I of $n=5$ real numbers $abcde$ is exactly equivalent to a DFT of eight real numbers $abcdedcb$ (even symmetry), here divided by two. (In contrast, DCT types II-IV involve a half-sample shift in the equivalent DFT.) Note, however, that the DCT-I is not defined for n less than 2. (All other DCT types are defined for any positive n .)

Thus, the DCT-I corresponds to the boundary conditions: x_k is even around $k=0$ and even around $k=n-1$; similarly for f_j .

DCT-II

$$f_j = \sum_{k=0}^{n-1} x_k \cos \left[\frac{\pi}{n} j \left(k + \frac{1}{2} \right) \right]$$

The DCT-II is probably the most commonly used form, and is often simply referred to as "the DCT".

Some authors further multiply the f_0 term by $1/\sqrt{2}$ (see below for the corresponding change in DCT-III). This makes the DCT-II matrix orthogonal (up to a scale factor), but breaks the direct correspondence with a real-even DFT of half-shifted input.

The DCT-II implies the boundary conditions: x_k is even around $k=-1/2$ and even around $k=n-1/2$; f_j is even around $j=0$ and odd around $j=n$.

DCT-III

$$f_j = \frac{1}{2}x_0 + \sum_{k=1}^{n-1} x_k \cos \left[\frac{\pi}{n} \left(j + \frac{1}{2} \right) k \right]$$

Because it is the inverse of DCT-II (up to a scale factor, see below), this form is sometimes simply referred to as "the inverse DCT" ("IDCT").

Some authors further multiply the x_0 term by $\sqrt{2}$ (see above for the corresponding change in DCT-II). This makes the DCT-III matrix orthogonal (up to a scale factor), but breaks the direct correspondence with a real-even DFT of half-shifted output.

The DCT-III implies the boundary conditions: x_k is even around $k=0$ and odd around $k=n$; f_j is even around $j=-1/2$ and odd around $j=n-1/2$.

DCT-IV

$$f_j = \sum_{k=0}^{n-1} x_k \cos \left[\frac{\pi}{n} \left(j + \frac{1}{2} \right) \left(k + \frac{1}{2} \right) \right]$$

The DCT-IV matrix is orthogonal (up to a scale factor).

A variant of the DCT-IV, where data from different transforms are *overlapped*, is called the modified discrete cosine transform (MDCT).

The DCT-IV implies the boundary conditions: x_k is even around $k=-1/2$ and odd around $k=n-1/2$; similarly for f_j .

DCT V-VIII

DCT types I-IV are equivalent to real-even DFTs of even order. In principle, there are actually four additional types of discrete cosine transform (Martucci, 1994), corresponding to real-even DFTs of logically odd order, which have factors of $n+1/2$ in the denominators of the cosine arguments. However, these variants seem to be rarely used in practice.

(The trivial real-even array, a length-one DFT (odd length) of a single number a , corresponds to a DCT-V of length $n=1$.)

Inverse transforms

The inverse of DCT-I is DCT-I multiplied by $2/(n-1)$. The inverse of DCT-IV is DCT-IV multiplied by $2/n$. The inverse of DCT-II is DCT-III multiplied by $2/n$ (and vice versa).

Like for the DFT, the normalization factor in front of these transform definitions is merely a convention and differs between treatments. For example, some authors multiply the transforms by $\sqrt{2/n}$ so that the inverse does not require any additional multiplicative factor.

Computation

Although the direct application of these formulas would require $O(n^2)$ operations, as in the fast Fourier transform (FFT) it is possible to compute the same thing with only $O(n \log n)$ complexity by factorizing the computation. (One can also compute DCTs via FFTs combined with $O(n)$ pre- and post-processing steps.)

References

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External link

- discrete cosine transform (*<http://planetmath.org/?op=getobj&from=objects&id=1469>*) on PlanetMath.

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