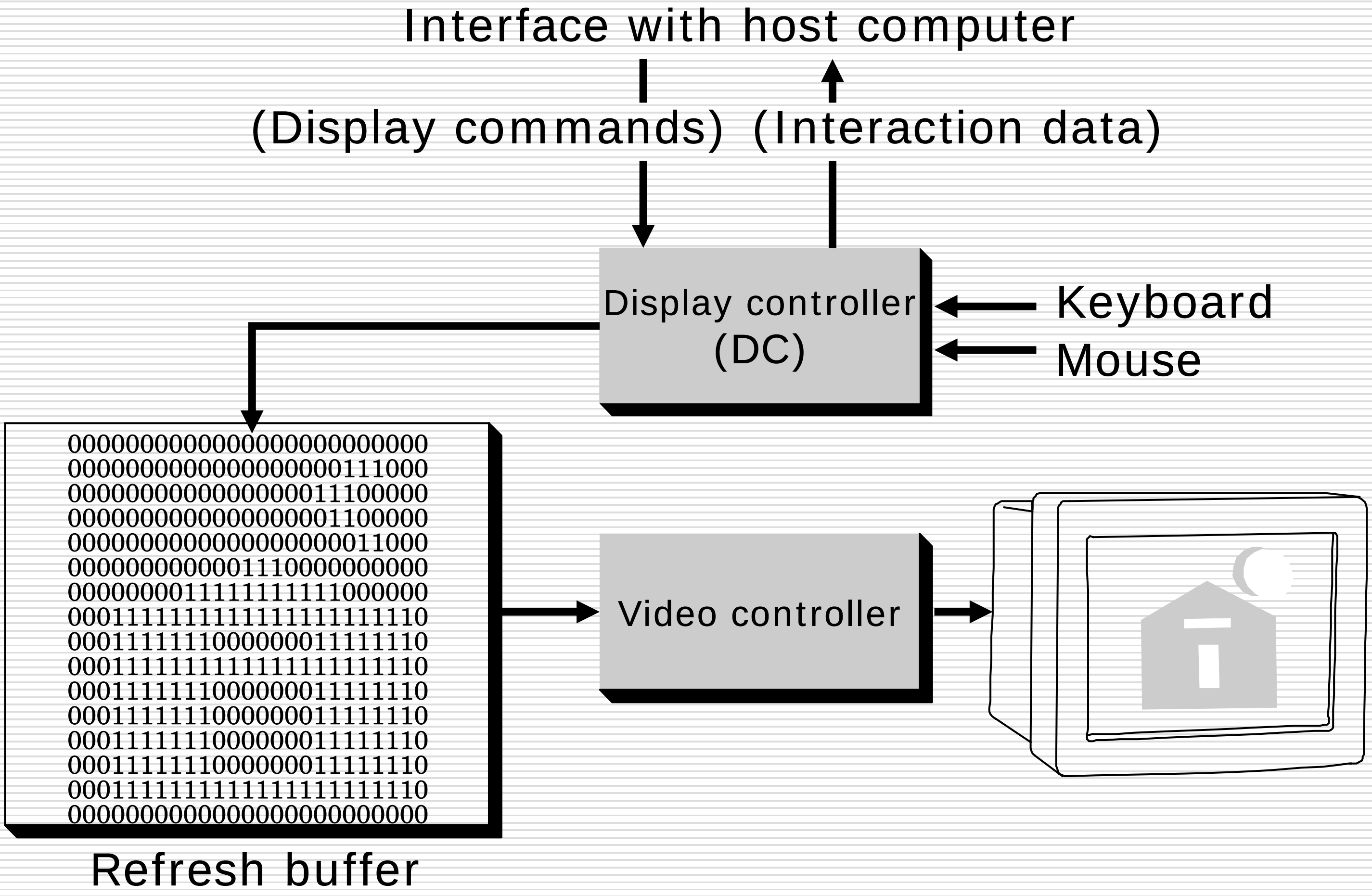
Computer Graphics

Bing-Yu Chen
National Taiwan University

Basic Raster Graphics Algorithms for Drawing 2D Primitives

- Architecture of a Raster Display
- Scan Converting Lines
- Filling Rectangles
- Filling Polygons
- Clipping Lines
- Clipping Polygons
- Antialiasing

Architecture of a Raster Display



```
0000000000000000000000000
0000000000000000000111000
0000000000000000011100000
0000000000000000001100000
0000000000000000000011000
0000000000001110000000000
0000000011111111111000000
0001111111111111111111110
0001111111000000011111110
0001111111111111111111110
0001111111000000011111110
0001111111000000011111110
0001111111000000011111110
0001111111000000011111110
0001111111111111111111110
0000000000000000000000000
```

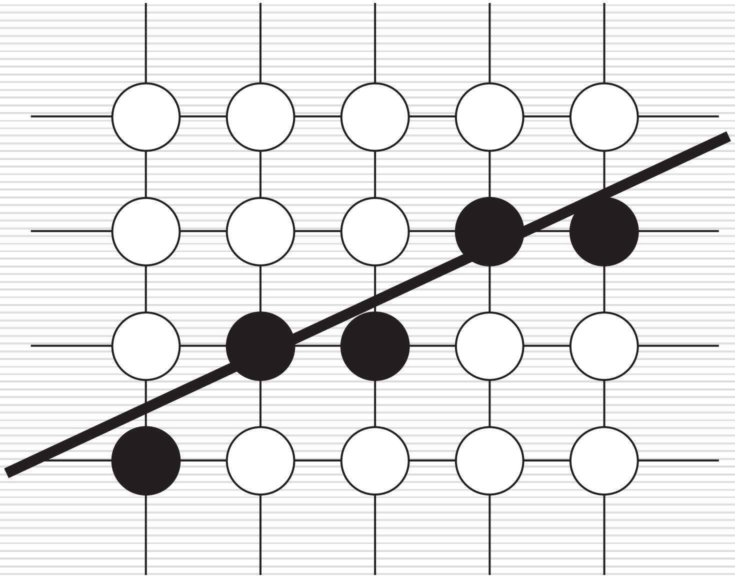
Definitions

- Pixel
  - a screen consists of N x M pixels
- Bilevel
  - = monochrome, 1 bit / pixel
- Color: RGB model
  - 16bits / pixel
    - R, G, B each 5 bits, 1 bit overlay
  - 24bits / pixel
    - R, G, B each 8 bits
  - 8 bits / pixel
    - 256 colors, color map, indexing

Definitions

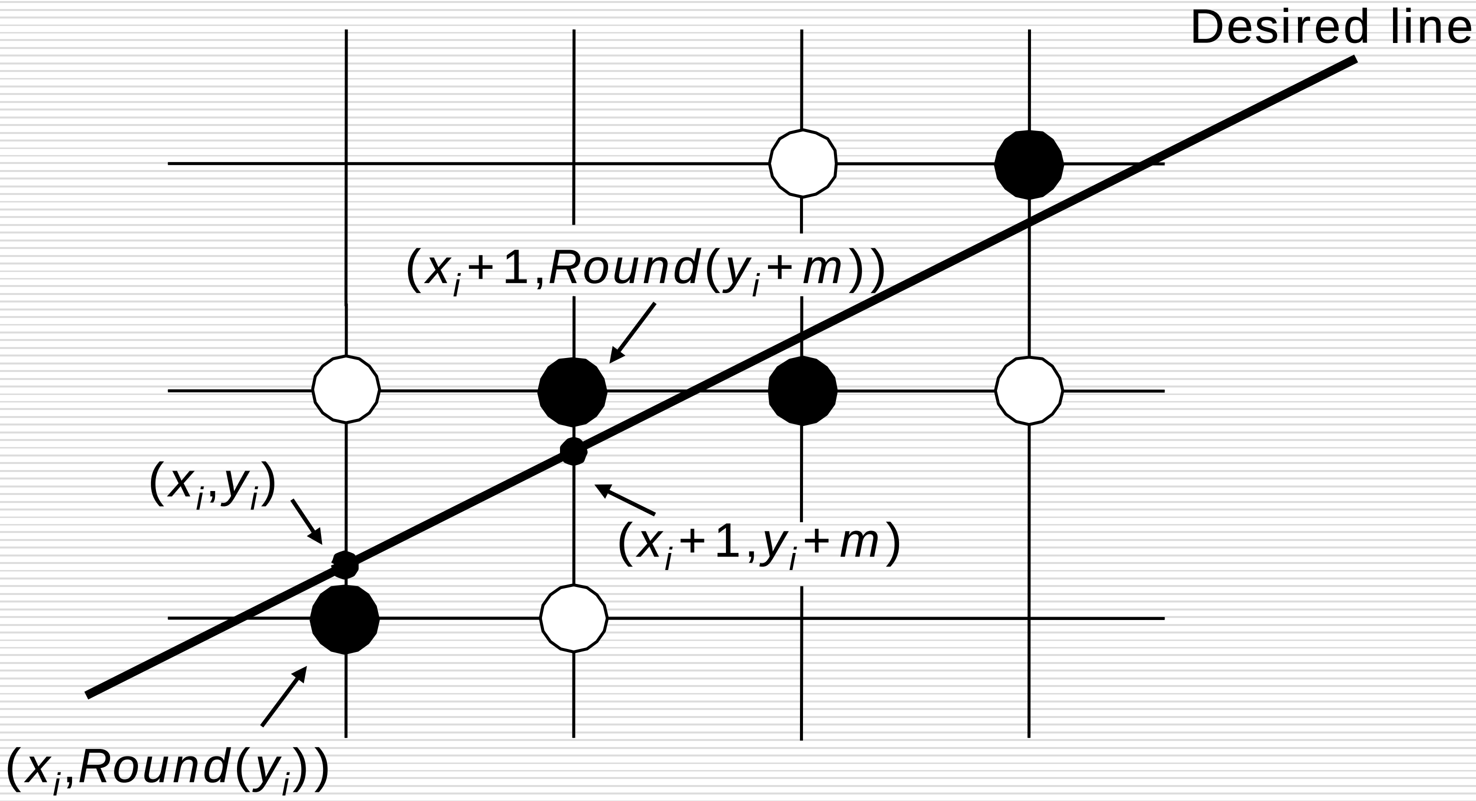
- bitmap / pixmap
  - bitmap
    - 1-bit-per-pixel bilevel systems
  - pixmap
    - multiple-bit-per-pixel systems
- frame buffer
  - an array of data in memory mapped to screen

Scan Converting Lines



- A scan-converted line showing intensified pixels as black circles

The Basic Incremental Algorithm



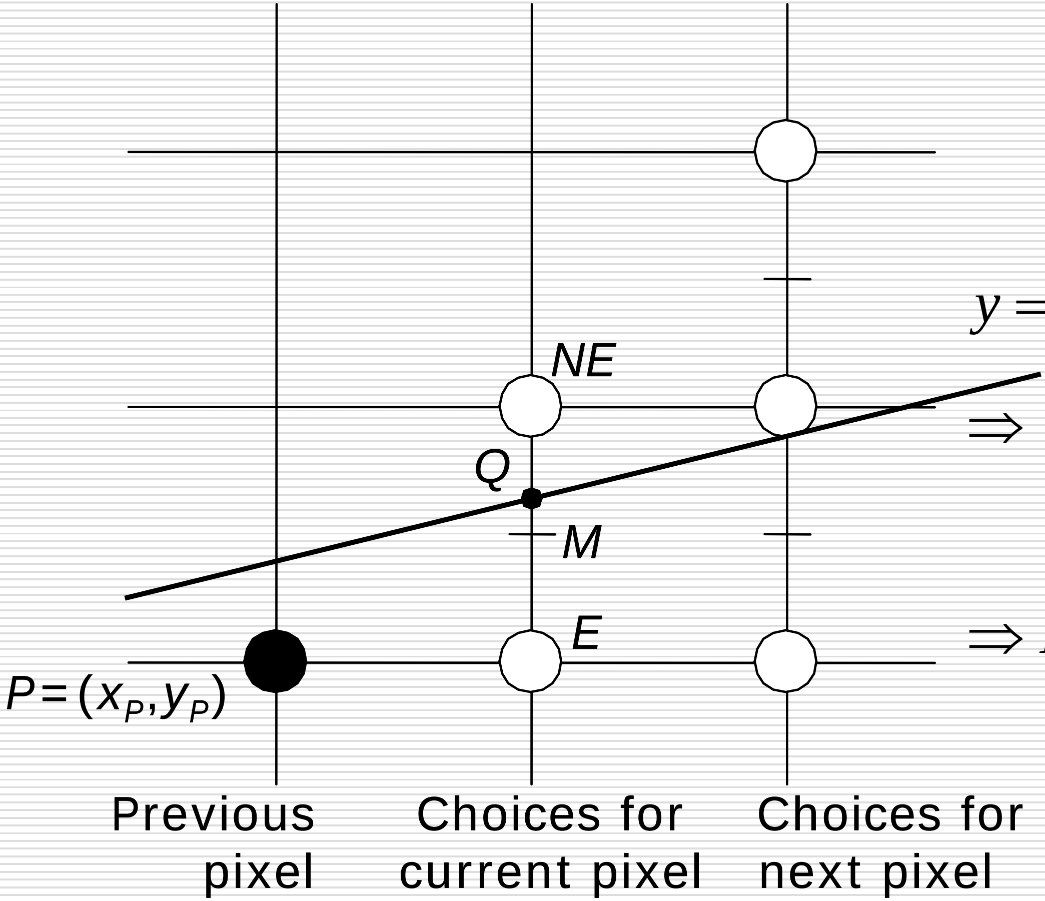
$$y_{i+1} = mx_{i+1} + B = m(x_i + \Delta x) + B = y_i + m\Delta x$$

The Basic Incremental Algorithm

```
void Line (int x0, int y0, int x1, int y1, value) {
    int x;
    float dy, dx, y, m;

    dy=y1-y0;
    dx=x1-x0;
    m=dy/dx;
    y=y0;
    for (x=x0; x<=x1; x++) {
        WritePixel (x, (int)floor(y+0.5), value);
        y+=m;
    }
}
```

Midpoint Line Algorithm



$$y = \frac{dy}{dx} x + B$$

$$\Rightarrow \quad F(x, y) = dy \cdot x - dx \cdot y + B \cdot dx = 0$$

$$= a \cdot x + b \cdot y + c$$

$$\Rightarrow F(M) = F(x_P + 1, y_P + \frac{1}{2}) = d$$

Midpoint Line Algorithm

$$d_{old} = F(x_P + 1, y_P + \frac{1}{2}) = a(x_P + 1) + b(y_P + \frac{1}{2}) + c$$

$$d_{new} = \begin{cases} F(x_P + 2, y_P + \frac{1}{2}) = a(x_P + 2) + b(y_P + \frac{1}{2}) + c & \text{for E} \\ F(x_P + 2, y_P + \frac{3}{2}) = a(x_P + 2) + b(y_P + \frac{3}{2}) + c & \text{for NE} \end{cases}$$

$$d_{new} = \begin{cases} d_{old} + a & \text{for E} \\ d_{old} + a + b & \text{for NE} \end{cases}$$

$$d_0 = F(x_0 + 1, y_0 + \frac{1}{2}) = a + \frac{b}{2} = dy - \frac{dx}{2}$$

Midpoint Line Algorithm

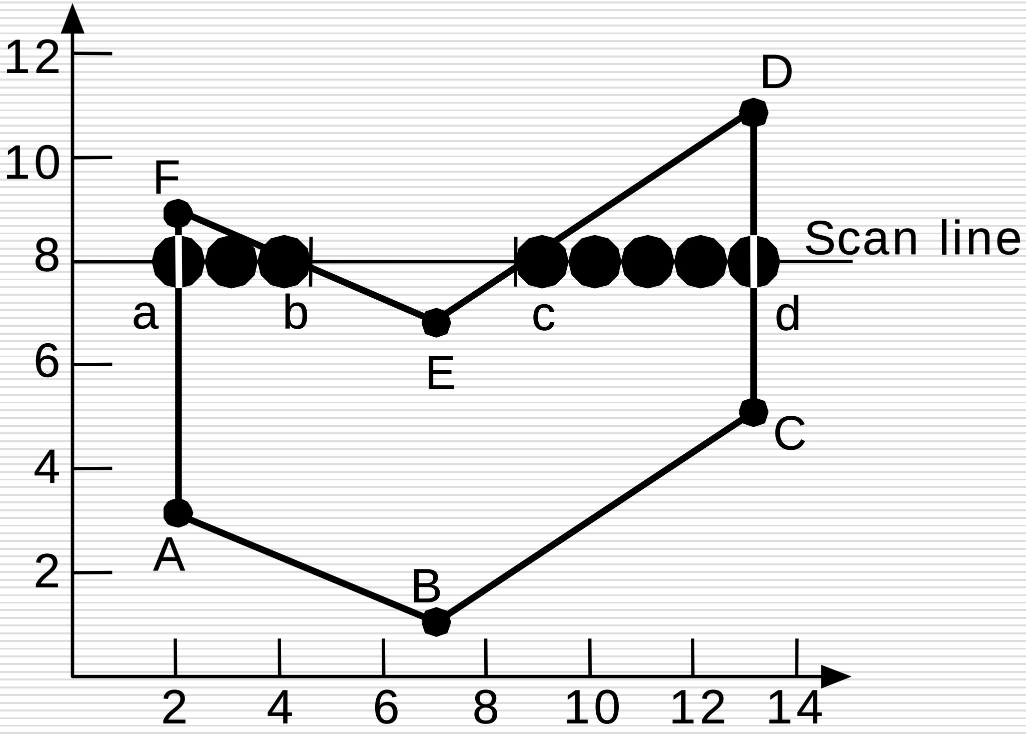
```
void MidpointLine (int x0, int y0, int x1, int y1, value) {
    int dx, dy, incrE, incrNE, d, x, y;

    dy=y1-y0;                   dx=x1-x0;                   d=dy*2-dx;
    incrE=dy*2;                 incrNE=(dy-dx)*2;
    x=x0;                       y=y0;
    WritePixel (x, y, value);
    while (x<x1) {
        if (d<=0) {             d+=incrE;         x++;
        } else {                d+=incrNE;        x++;      y++;
        }
        WritePixel (x, y, value);
    }
}
```

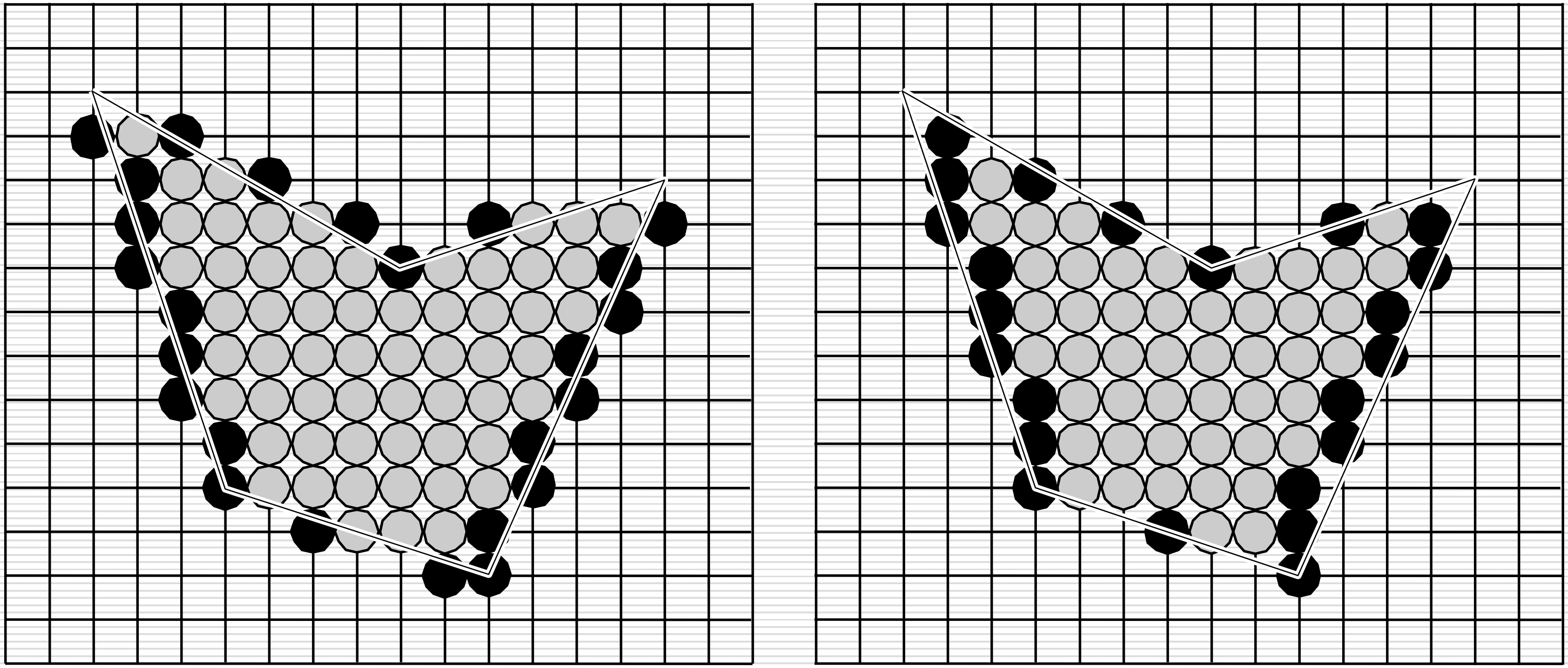
Filling Rectangles

```
for (y from ymin to ymax of the rectangle) {
    for (x from xmin to xmax) {
        WritePixel (x, y, value);
    }
}
```

Filling Polygons



Filling Polygons



● Span extrema ○ Other pixels in the span

Filling Polygons

1. find the intersections of the scan line with all edges of the polygon
2. sort the intersections by increasing *x* coordinate
3. fill in all pixels between pairs of intersections that lie interior to the polygon

Step 3 requires 4 elaborations

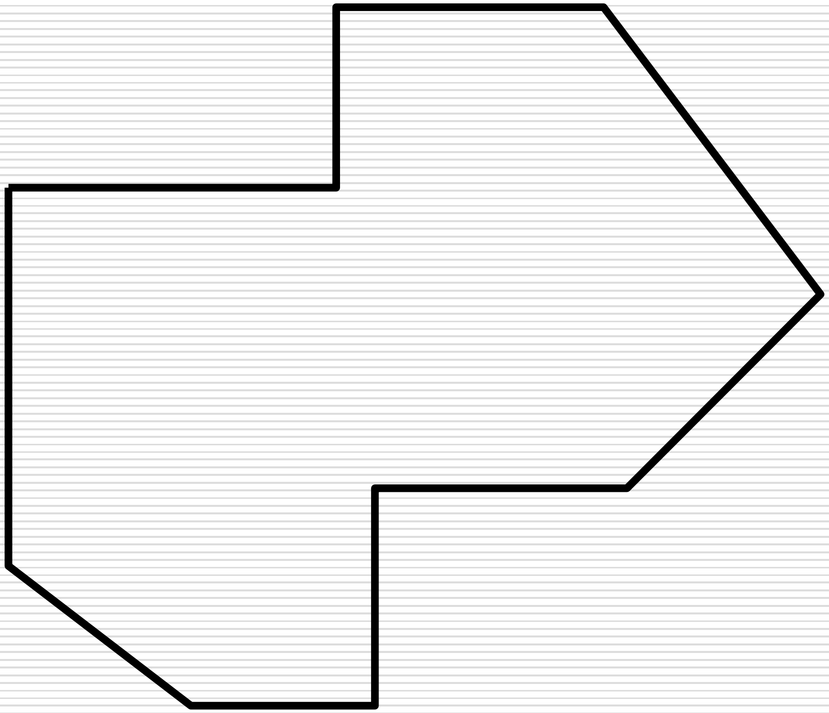
3.1 given an intersection with an arbitrary, fractional x value, how do we determine which pixel on either side of that intersection is interior ?

3.2 how do we deal with the special case of intersections at integer pixel coordinates ?

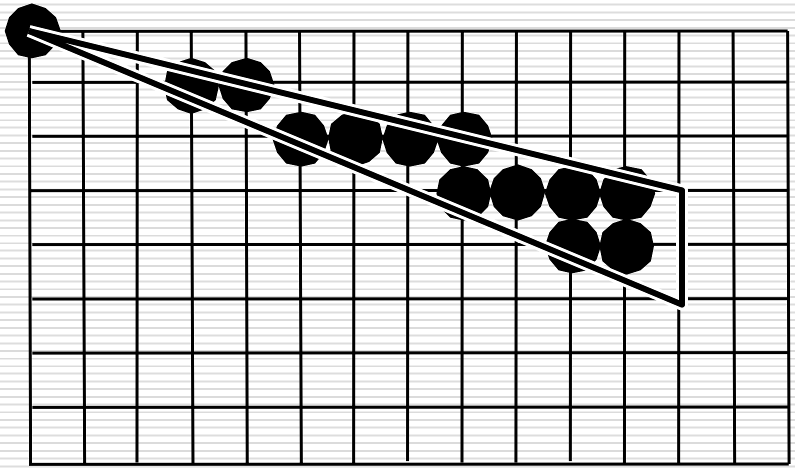
3.3 how do we deal with the special case in step 3.2 for shared vertices ?

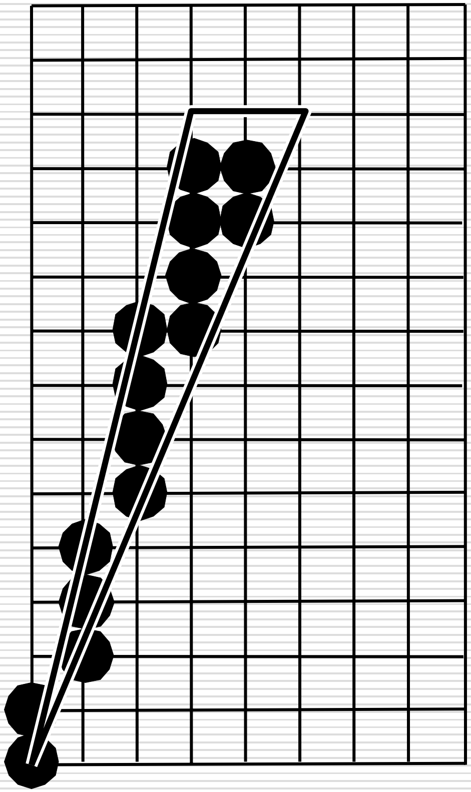
3.4 how do we deal with the special case in step 3.2 in which the vertices define a horizontal edge ?

Horizontal Edges

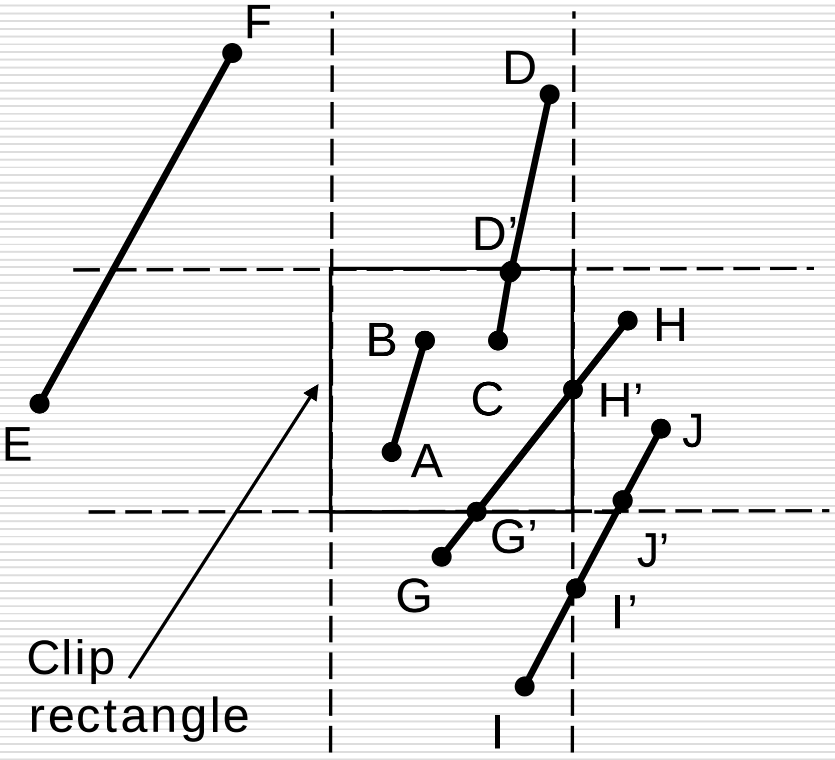


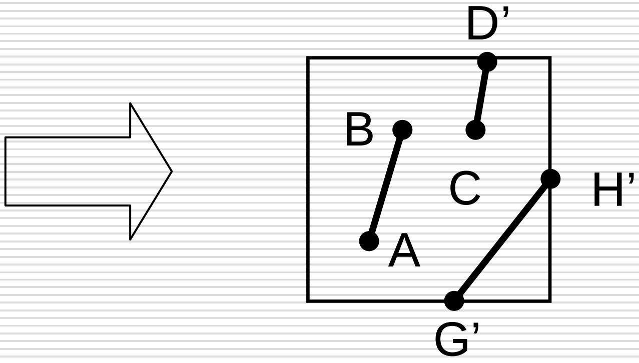
Slivers





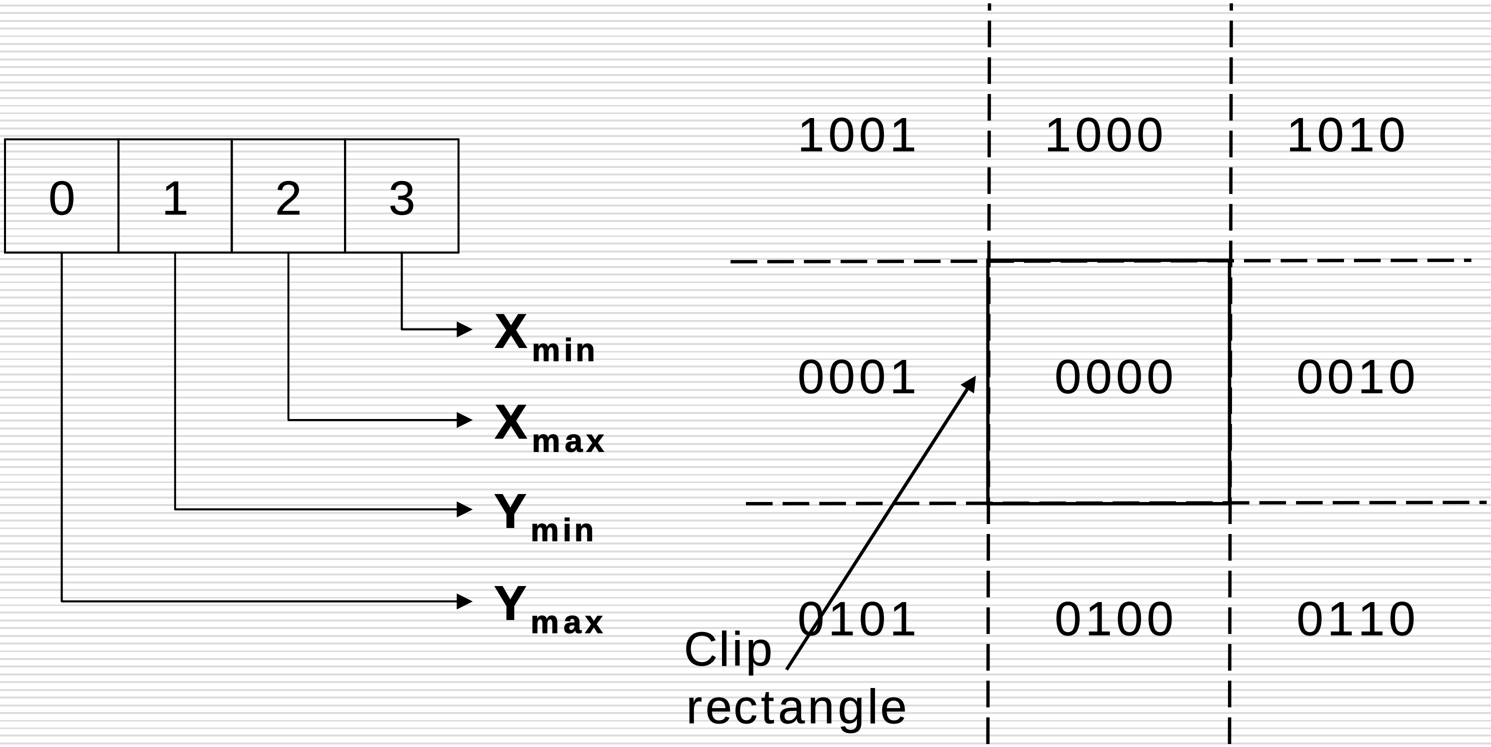
Clipping Lines





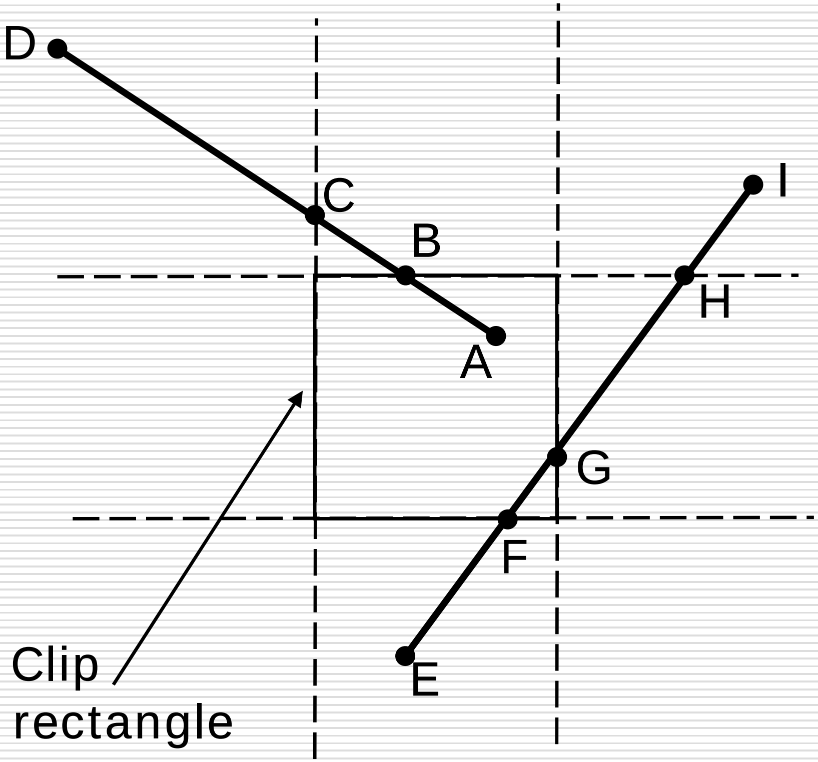
$$x = x_0 + t(x_1 - x_0)$$
$$y = y_0 + t(y_1 - y_0)$$

The Cohen-Sutherland Line-Clipping Algorithm

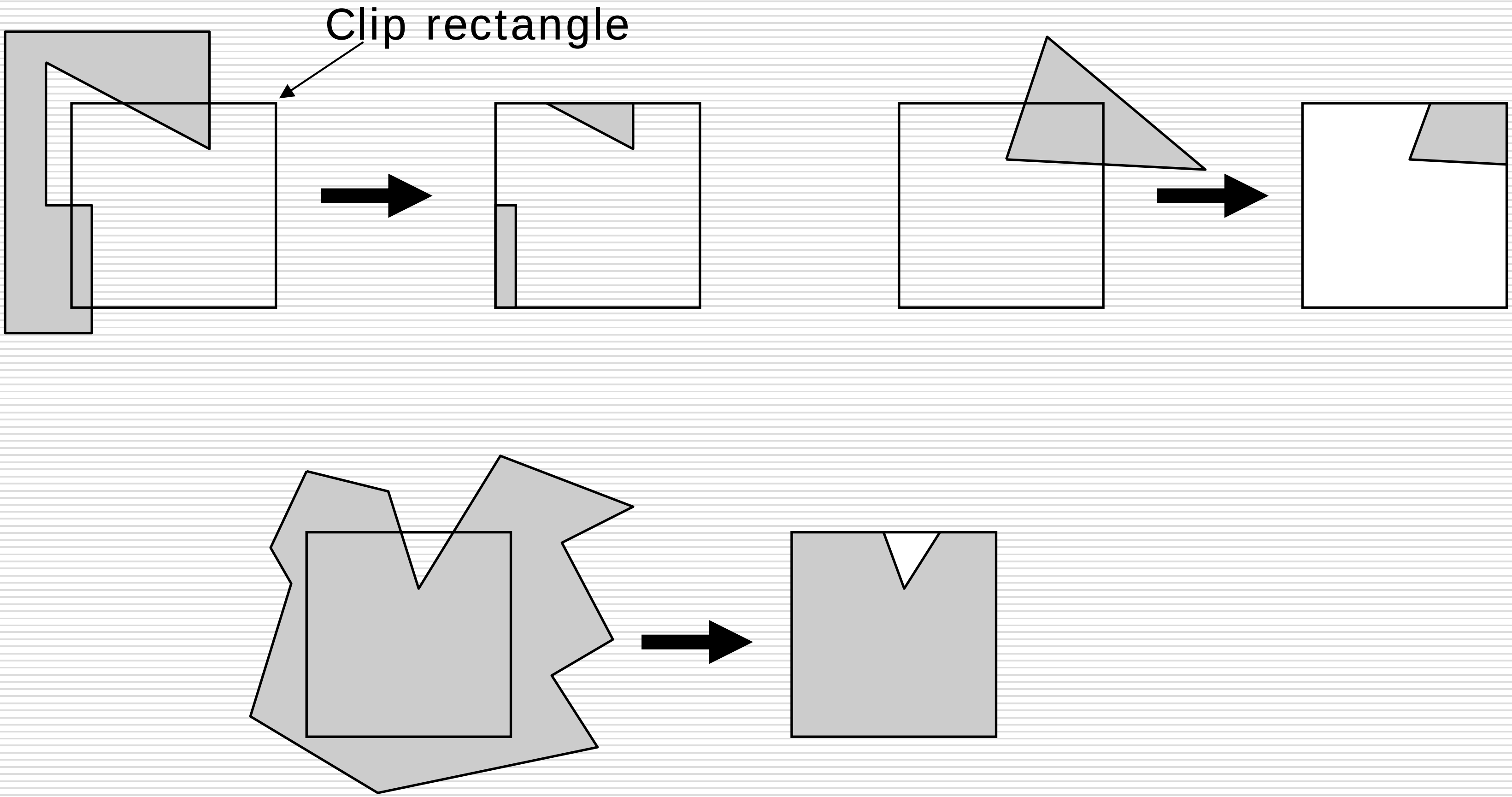


1001	1000	1010
0001	0000	0010
0101	0100	0110

The Cohen-Sutherland Line-Clipping Algorithm

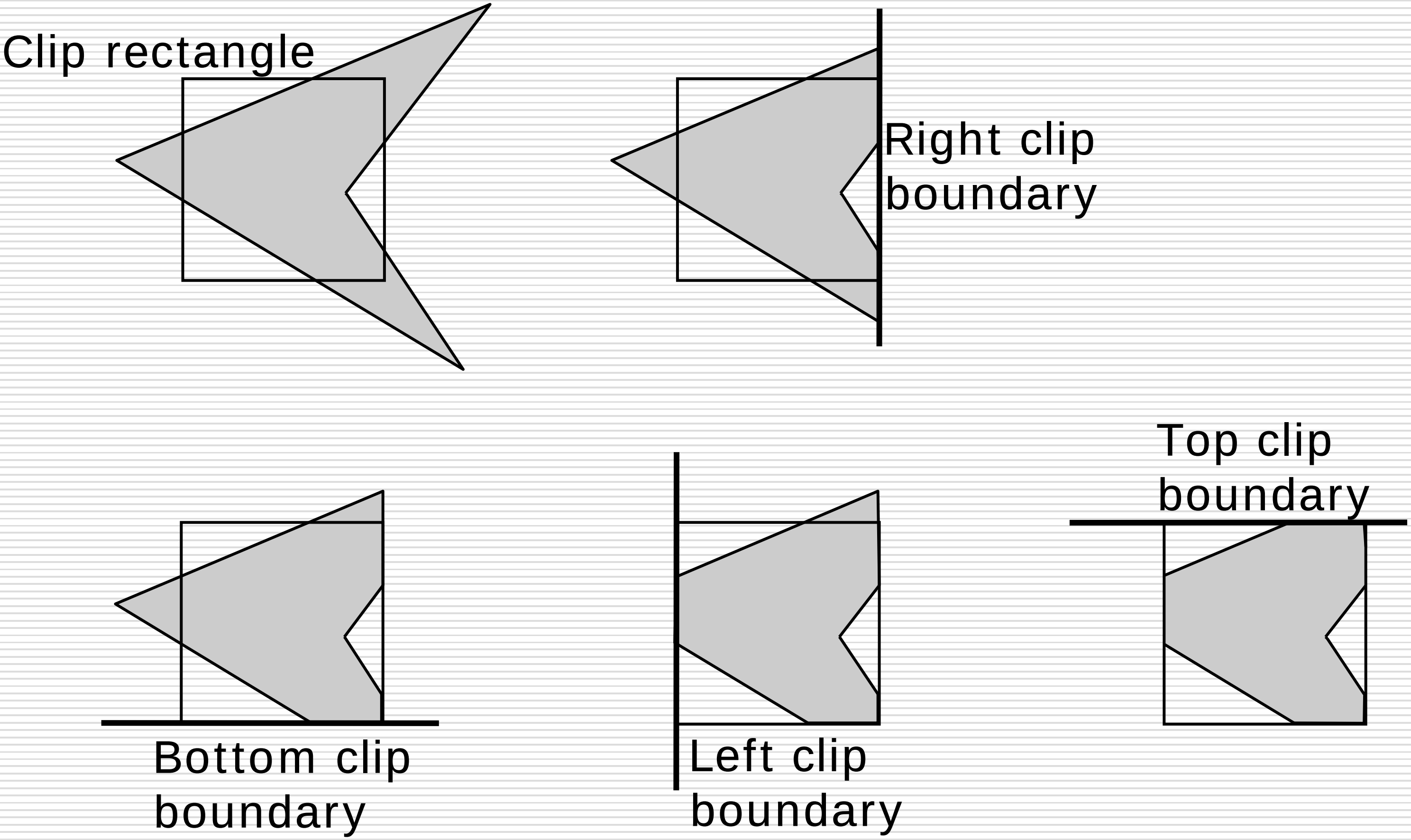


Clipping Polygons



Clip rectangle

The Sutherland-Hodgman Polygon-Clipping Algorithm



Clip rectangle

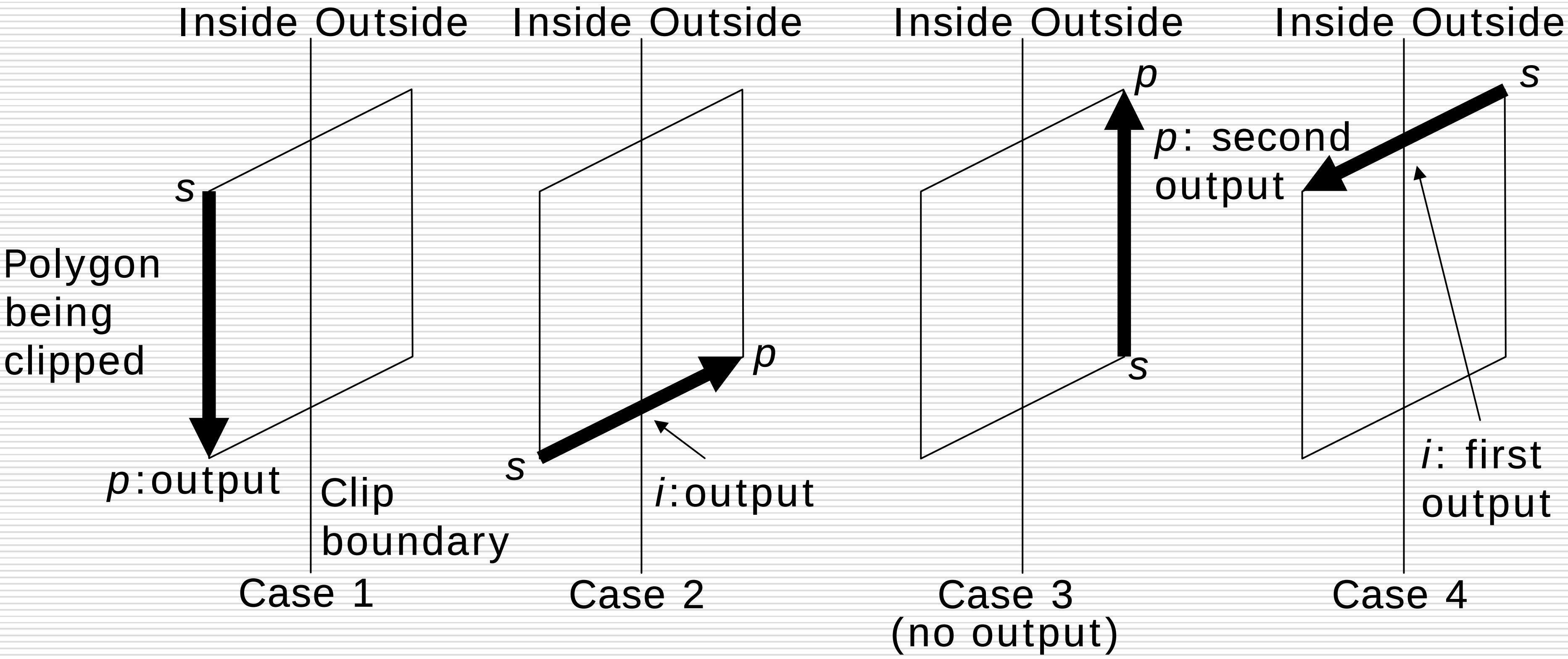
Right clip boundary

Bottom clip boundary

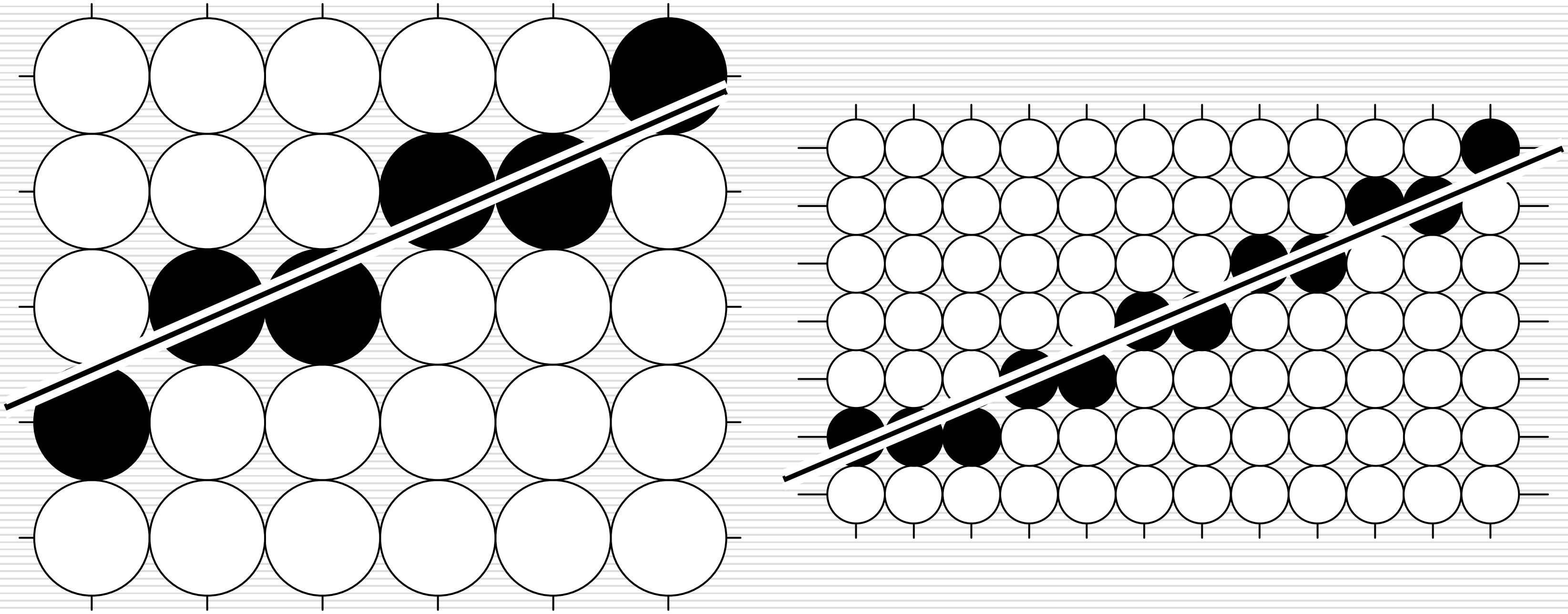
Left clip boundary

Top clip boundary

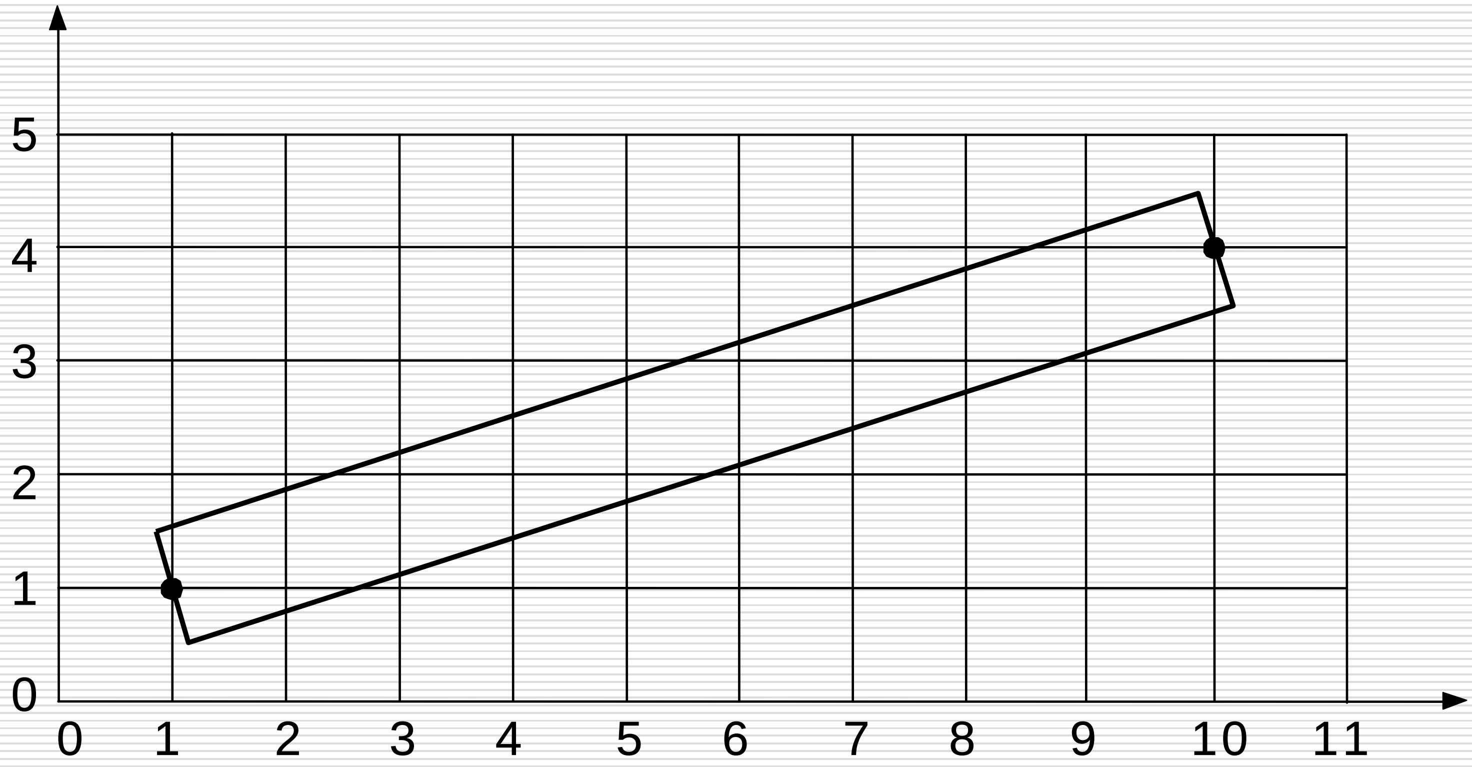
The Sutherland-Hodgman Polygon-Clipping Algorithm



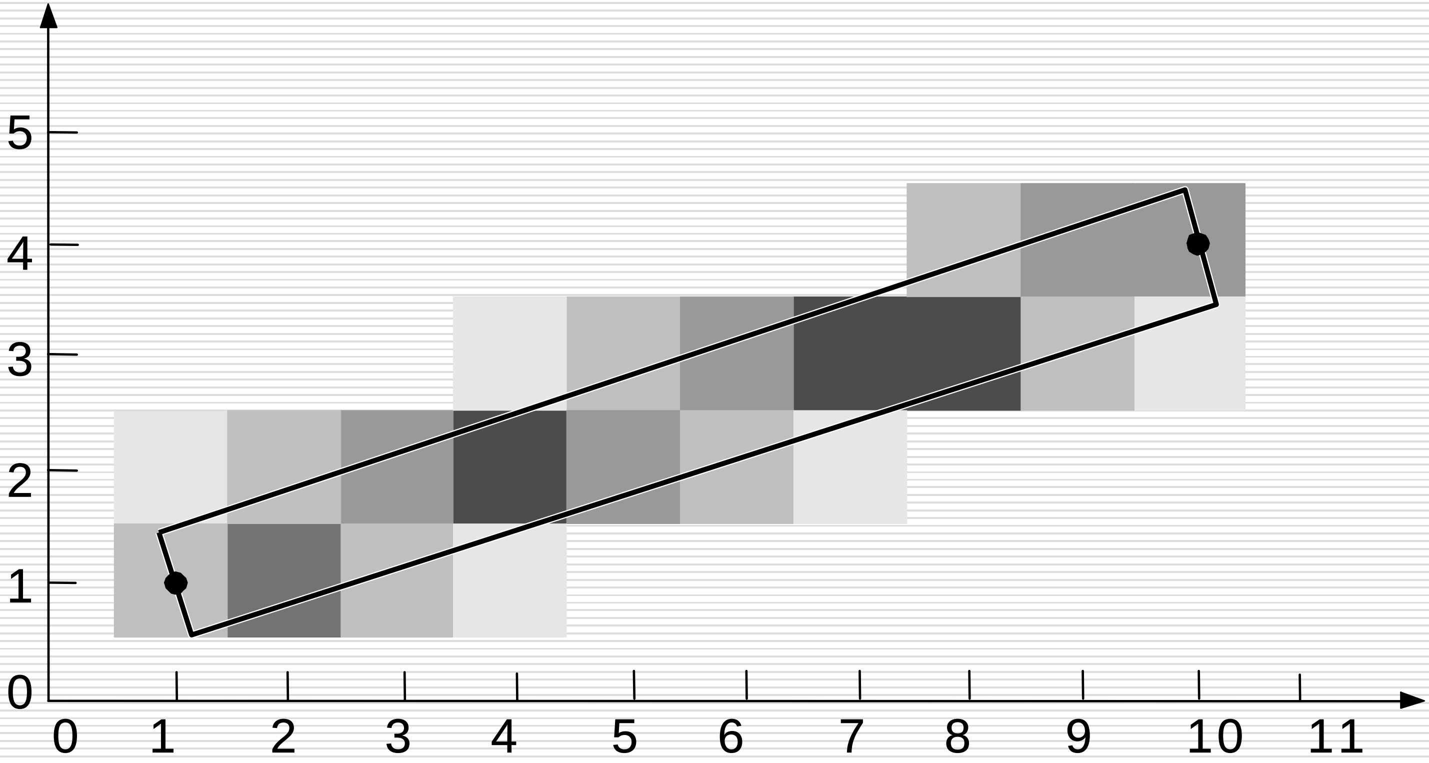
Antialiasing



Unweighted Area Sampling



Unweighted Area Sampling



5
4
3
2
1
0
0
1
2
3
4
5
6
7
8
9
10
11