Mathematical Analysis of Algorithms

**Homework #1**
**Due Date:**
**Reading Assignment:** Preface, Chapter 1, 2.1, 2.2
**Problems:**

**1.** 1–9

**2.** 1–16

**3.** 2–11

**4.** 2–22

**5.** Prove or disprove that the *Knuth Sequence* defined by

$$\begin{aligned} K(0) &= 1; \\ K(n+1) &= 1 + \min\left(2K(\lfloor\frac{n}{2}\rfloor), 3K(\lfloor\frac{n}{3}\rfloor)\right), \quad \text{for } n \geq 0, \end{aligned}$$

has the property that $K(n) \geq n$, for $n \geq 0$.
(Note that the sequence begins $1, 3, 3, 4, 7, 7, 7, 9, 9, 10, 13, \ldots$ )

**6.** Consider the series of fractions

$$\frac{1}{2}, \frac{1/2}{3/4}, \frac{\frac{1}{2}/\frac{3}{4}}{\frac{5}{6}/\frac{7}{8}}, \frac{\frac{1/2}{3/4}/\frac{5/6}{7/8}}{\frac{9/10}{11/12}/\frac{13/14}{15/16}}, \ldots$$

Suppose that each fraction is simplified to be a fraction of two products of integers (for example, the third is $\frac{1\cdot4\cdot6\cdot7}{2\cdot3\cdot5\cdot8}$.) Prove that, for the $n^{\text{th}}$ fraction, the sum of the $k^{\text{th}}$ powers of the numbers in the numerator equals the sum of the $k^{\text{th}}$ powers of the numbers in the denominator for $0 \leq k < n$. (For example, $1^2 + 4^2 + 6^2 + 7^2 = 2^2 + 3^2 + 5^2 + 8^2$ )

**Extra credit**: What limit do the values of these fractions approach?