Mathematical Analysis of Algorithms

Homework #8
Due Date:
Reading Assignment: 8.1–8.4
Problems:

1. We roll a pair of dice n times, and compute the sum of all $2n$ faces which come up. Suppose each roll of each dice is independent of the other rolls.

(a) What is the expected value of sum?

(b) What is the variance?

(c) How many rolls are sufficient to ensure, with probability 99%, that the sum is greater than 100?

2. 8–8

3. 8–15

4. 8–24

5. 8–29

6. Let h and k be two relatively prime integers greater than 1. We define $N(h,k)$ to be the number of positive integers that **cannot** be expressed in the form $ih + jk$, for nonnegative integers i and j, namely,

$$N(h,k) = \left| \left\{ n \in \mathbb{N} \mid n \neq ih + jk, \text{for all } i, j = 0, 1, 2, \ldots \right\} \right|$$

Express $N(h,k)$ in closed form as a function of h and k.

Hint: every integer $n \geq hk$ can be expressed in the form $ih + jk$.
Motivation: (You are not responsible for this) The value $N(h,k)$ is the number of comparisons (in the worst case) required to insert an element of the array into its proper place during the last phase of $(h,k,1)$ **ShellSort**. (see Knuth, Vol. 3)

The first phase of **ShellSort** consists of performing the h insertion sorts on the subarrays

$$\begin{gathered}\{X(1), X(h+1), X(2h+1), \ldots\} \\ \{X(2), X(h+2), X(2h+2), \ldots\} \\ \vdots \\ \{X(h), X(2h), X(3h), \ldots\}\end{gathered}$$

The second phase consists of performing the k insertion sorts on the subarrays

$$\begin{gathered}\{X(1), X(k+1), X(2k+1), \ldots\} \\ \{X(2), X(k+2), X(2k+2), \ldots\} \\ \vdots \\ \{X(k), X(2k), X(3k), \ldots\}\end{gathered}$$

After these two phases, any two elements $ih + jk$ slots apart in the array are already sorted. (This isn't obvious!)

The final phase of **ShellSort** consists of a single insertion sort on the entire array. The maximum number of slots each element must "move over" during the last phase is $N(h, k)$.