1. Given the following three functions:

a. A 2-bit-wide shifter takes two input signals, \( i_0 \) and \( i_1 \), and shifts them to two outputs, \( o_0 \) and \( o_1 \), under the control of a shift signal. If this signal \( \text{SHIFT} \) is false, then the inputs are connected straight through to the outputs. If \( \text{SHIFT} \) is true, then \( i_0 \) is routed to \( o_1 \) and \( o_0 \) should be set to a 0.

b. A 1-bit demultiplexer takes an input signal \( \text{IN} \) and shifts it to one of two outputs, \( o_0 \) and \( o_1 \), under the control of a single \( \text{SELECT} \) signal. If \( \text{SELECT} \) is 0, then \( \text{IN} \) is connected through to \( o_0 \) and \( o_1 \) is connected to a 0. If \( \text{SELECT} \) is 1, then \( \text{IN} \) is connected through to \( o_1 \) and \( o_0 \) is connected to a 0.

c. A 2-bit multiplexer takes two input signals, \( i_0 \) and \( i_1 \), and shifts one of them to the single output \( \text{OUT} \) under the control of a 1-bit select signal. If the \( \text{SELECT} \) signal is false, then \( i_0 \) is passed to \( \text{OUT} \). If \( \text{SELECT} \) is true, then \( i_1 \) is passed to \( \text{OUT} \).

Complete the following five items:

a. Construct their truth tables.

b. What are the functions in sum of products forms, using “little \( m \)” notation?

c. What are the functions in product of sums forms, using “big \( M \)” notation?

d. Use the Karnaugh map method to simplify the functions in sum of products forms.

e. Draw logic schematics using AND, OR, and INVERT gates.

2. Simplify the following functions using the theorems of Boolean algebra. Write the particular law or theorem you are using in each step. For each simplified function you derive, how many literals does it have?

a. \( f(X, Y) = XY + X\overline{Y} \)

b. \( f(X, Y) = (X + Y)(X + \overline{Y}) \)

c. \( f(X, Y, Z) = \overline{X}\overline{Y}Z + X\overline{Y}Z + XYZ \)

d. \( f(X, Y, Z) = (X + Y)(\overline{X} + Y + Z)(\overline{X} + Y + \overline{Z}) \)

e. \( f(W, X, Y, Z) = X + XYZ + \overline{X}YZ + \overline{X}Y + WX + \overline{W}X \)