Hierarchical Mesh Decomposition using Fuzzy Clustering and Cuts

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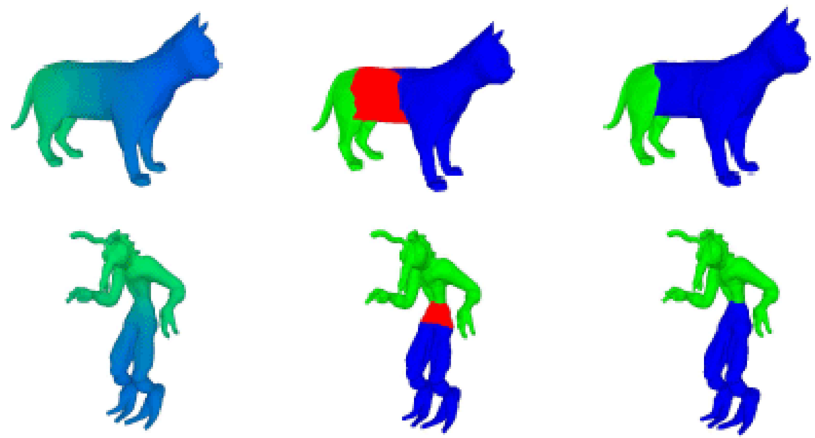
- Introduction
- Algorithm – binary case
- Algorithm – k way case
- Result
- Conclusion

Introduction

- Fuzzy decomposition



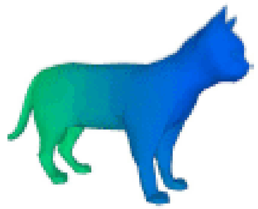
(a) probabilities (b) fuzzy decomposition (c) decomposition

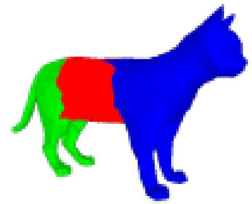
Definition 2.1 ***k-way Decomposition:*** *$S_1, S_2, \ldots S_k$ is a k-way decomposition of S iff (i) $\forall i, 1 \leq i \leq k$, $S_i \subseteq S$, (ii) $\forall i$, S_i is connected, (iii) $\forall i \neq j, 1 \leq i, j \leq k$, S_i and S_j are face-wise disjoint and (iv) $\cup_{i=1}^{k} S_i = S$.*

Definition 2.2 ***Binary Decomposition:*** *S_1, S_2 is a binary decomposition of S if it is a $k-$way decomposition with $k = 2$.*

Definition 2.3 ***Patch:*** *Given $S_1, S_2, \ldots S_k$, a k-way decomposition of S, each S_i is called a patch of S.*

Binary case (1/4)





- Assign Probability
  - Choose two faces REP_A and REP_B with largest distance as initial representative of the two cluster
  - Assign probabilities to every faces according to the distances to the initial patches:

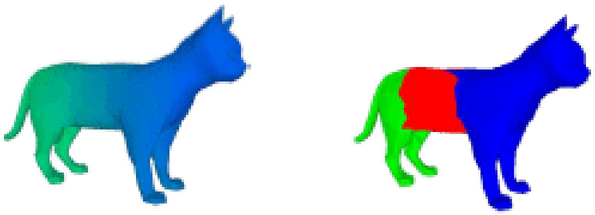
$$P_B(f_i) = \frac{Dist(f_i, REP_A)}{Dist(f_i, REP_A) + Dist(f_i, REP_B)} \qquad P_A(f_i) = 1 - P_B(f_i)$$

  - $Dist(f_i,f_j)$ is the shortest path from f_i to f_j in mesh's dual graph with

$$Weight(dual(f_i), dual(f_j)) = \delta \cdot \frac{Geod(f_i, f_j)}{avg(Geod)} + (1-\delta) \cdot \frac{Ang\_Dist(\alpha_{ij})}{avg(Ang\_Dist)}$$

$$Ang\_Dist(\alpha_{ij}) = \eta(1 - \cos\alpha_{ij})$$

Binary case (2/4)



- Generating fuzzy decomposition
  - Goal: cluster faces by minimize the function

$$F = \sum_{p} \sum_{f} probability(f \in patch(p)) \cdot Dist(f, p)$$

  - Algorithm
    1. Compute the probabilities of faces to belong to each patch
    2. Re-compute the set of representatives to minimize F by

$$REP_A = \min_f \sum_{f_i} (1 - P_B(f_i)) \cdot Dist(f, f_i)$$

$$REP_B = \min_f \sum_{f_i} P_B(f_i) \cdot Dist(f, f_i)$$

    3. If the representative set is changed, go back to 1
    4. If no change, partition faces as

$$A = \{ f_i \mid P_B(f_i) < 0.5 - \varepsilon \}$$

$$B = \{ f_i \mid P_B(f_i) > 0.5 + \varepsilon \}$$

$$C = \{ f_i \mid 0.5 - \varepsilon \le P_B(f_i) \le 0.5 + \varepsilon \}$$

Binary case (3/4)

- Generate final decomposition
  - G(V,E) : dual graph of mesh.
  - VA,VB : the set of dual vertices of patches A and B respectively
  - Goal: partition V into VA', VB' such that

$$V_A \subseteq V_{A'}, V_B \subseteq V_{B'}$$

$$weight(Cut(V_{A'}, V_{B'})) = \sum_{u \in V_{A'}, v \in V_{B'}} \omega(u,v) \text{ is minimal}$$

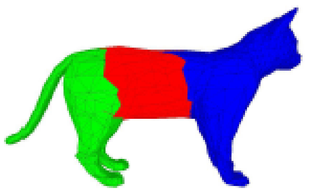
➔Construct a flow network graph G(V',E') such that

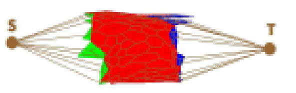
$$V' = V_C \cup V_{CA} \cup V_{CB} \cup \{S,T\}$$

$$E' = E_C \cup \{(S,v), \forall v \in V_{CA}\} \cup \{(T,v), \forall v \in V_{CB}\} \cup \{e_{ij} \in E \mid i \in V_C, j \in \{V_{CA} \cup V_{CB}\}\}$$

With capacity Cap(I,j) and use max flow algorithm to find min cut

$$Cap(i,j) = \begin{cases} \dfrac{1}{1 + \dfrac{Ang\_Dist(\alpha_{ij})}{avg(Ang\_Dist)}} & \text{if } \{\text{i, j} \neq S,T\} \\ \infty & \text{else} \end{cases}$$





Binary case (4/4)

- Hierarchy decomposition
  - Every patches can be recursively decomposed so that we can get a hierarchy structure
- Stop condition

The hierarchy decomposition is stopped when

(a) Distance between representative is smaller than a threshold

(b) Difference between max and min dihedral angle is smaller than threshold

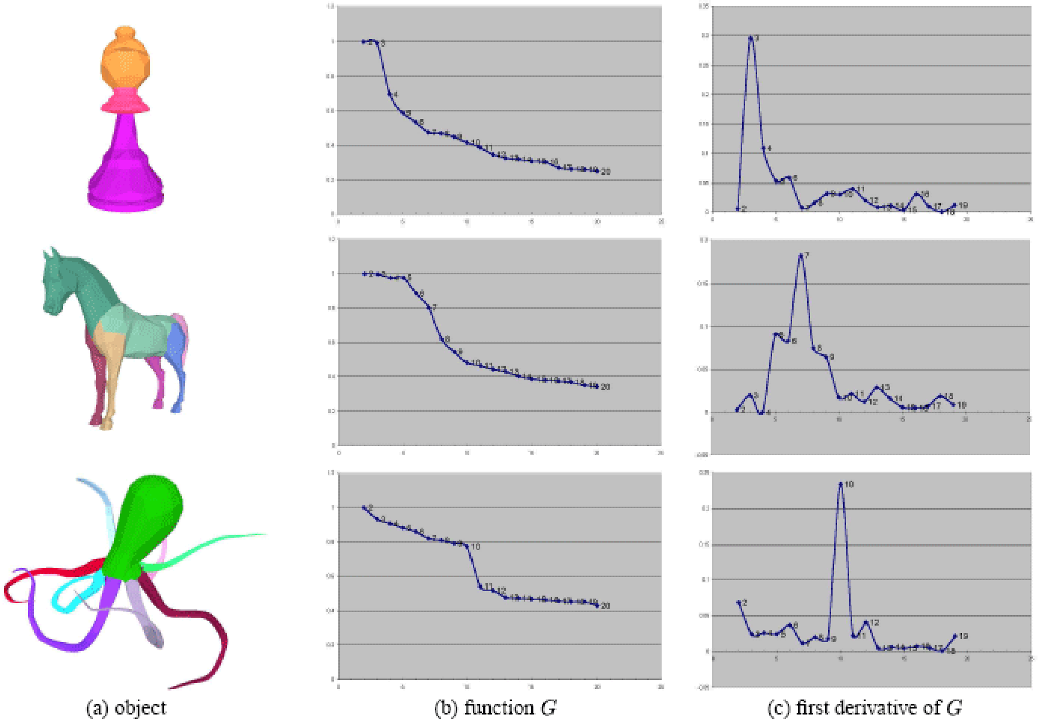
(c) The ratio between the average distance in the patch and that of the overall object does not exceed a threshold

K way case(1/3)

- A generalization of binary case
- Representatives are choose iteratively
  - 1st representative: Choose the face having the minimal sum of distances from all other faces (body)
  - Other representative: added in turn so as to maximize their minimal distance from previous assigned representatives

K way case(2/3)

- The number of representatives is chosen to minimize the first derivative of $G(k) = \min_{i<k}(Dist(REP_k, REP_i))$



(a) object (b) function G (c) first derivative of G

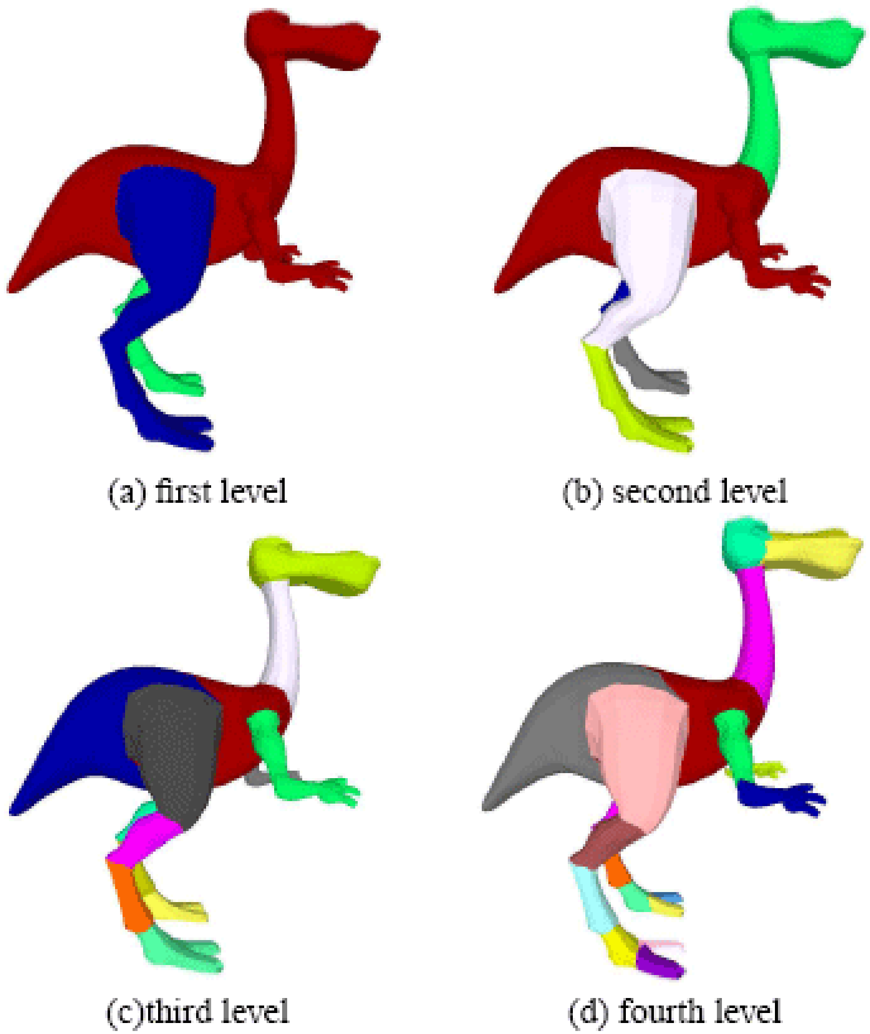
K way case(3/3)

- The probability that face f_i belonging to patch P_j is defined as

$$P_{P_j}(f_i) = \frac{\frac{1}{Dist(f_i, REP(P_j))}}{\sum_l \frac{1}{Dist(f_i, REP(P_l))}}$$

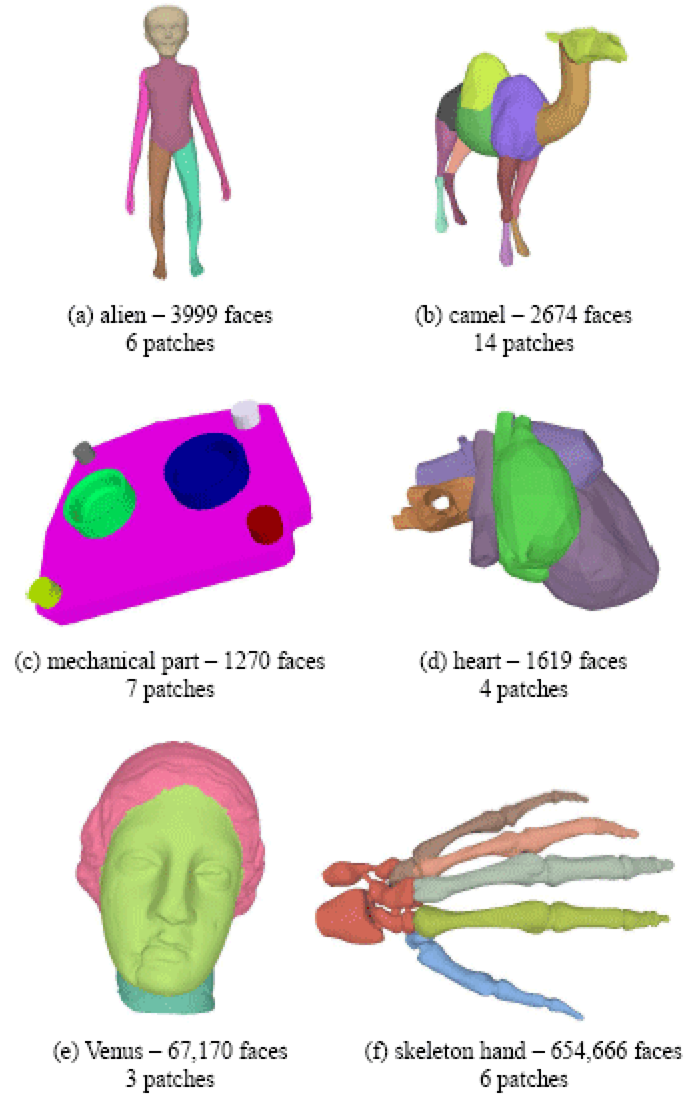
- To extract the fuzzy area, each pair of neighboring components are proceeded similarly to the binary case
- Complexity: $O(V^2 \log V + IV^2)$
  - V is number of vertex, I is number of iteration
  - $V^2\log V$ for all pair shortest path and minimum cut algorithm
  - IV^2 for assigning faces to patches

Result



(a) first level (b) second level

(c)third level (d) fourth level



(a) alien – 3999 faces
6 patches

(b) camel – 2674 faces
14 patches

(c) mechanical part – 1270 faces
7 patches

(d) heart – 1619 faces
4 patches

(e) Venus – 67,170 faces
3 patches

(f) skeleton hand – 654,666 faces
6 patches

Conclusion

- The hierarchically decomposition algorithm avoid jaggy boundaries as well as over segmentation
- Different distance function and capacity function can be experimented with
- Non-geometric features such as color and texture can be embedded in the algorithm

Thank you