

Inverse Kinematics for Reduced Deformable Models

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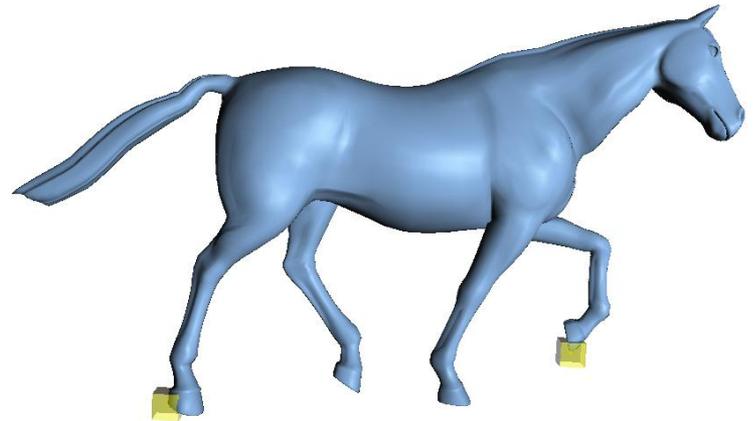
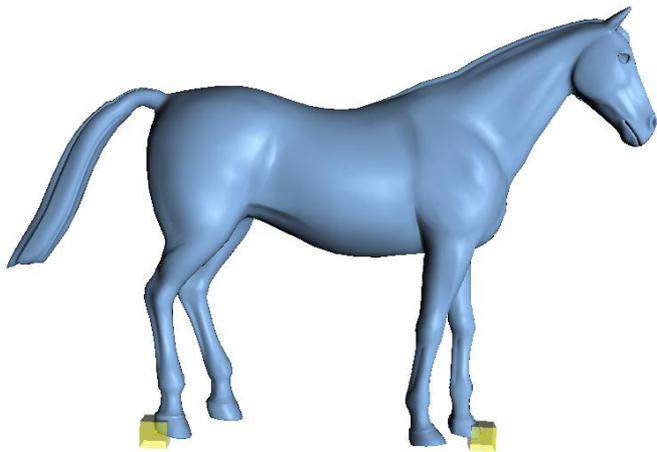
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Our Goal: Interaction

- mesh manipulation
- direct, intuitive control
- speed



Related Work

- mesh editing

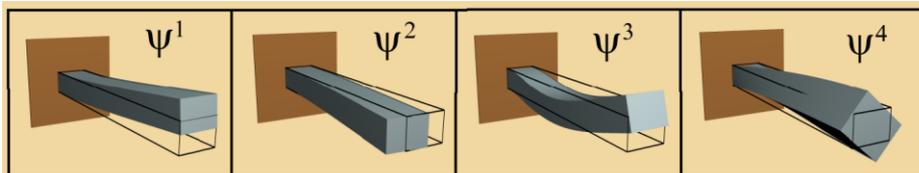
[Zorin et al. 1997, Kobbelt et al. 1998, Sorkine 2005]

- inverse kinematics

[Sumner et al. 2005, Grochow et al. 2003]

- reduced deformable models

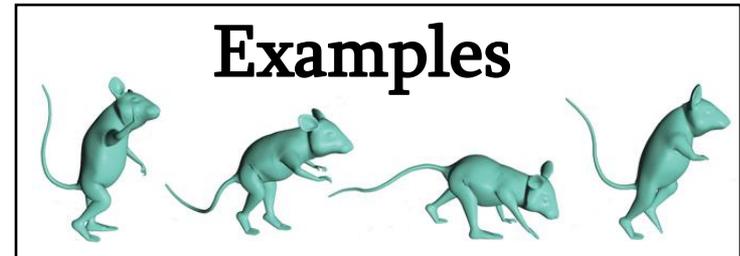
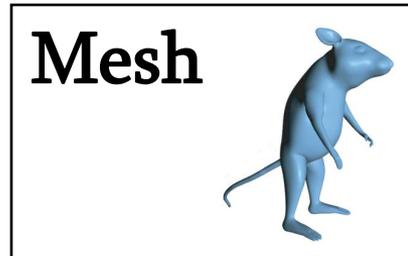
[Pentland and Williams 1989, Alexa and Muller 2000, James and Twigg 2005]



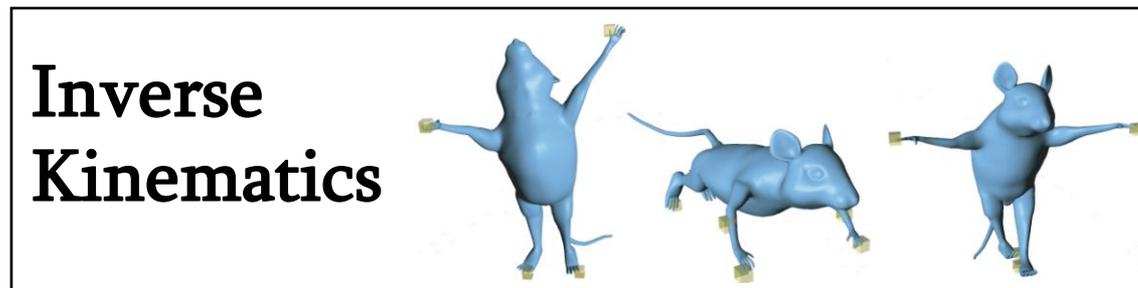
[James and Barbic 2005]



Our Method



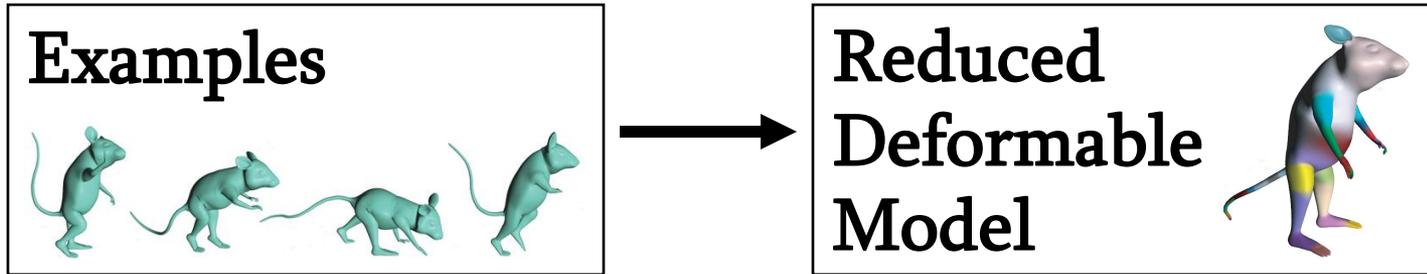
decouple deformation complexity
from geometric complexity!



- completely automated animation pipeline



Reduced Deformable Model



[James and Twigg 2005]

- expresses deformations compactly
- automatically constructed

Control parameters: $\{ \mathbf{F}_i, \mathbf{d}_i \}$

- non-rigid
- no hierarchy



RDM: Mesh Reconstruction

Control parameters: $\{ \mathbf{F}_i, \mathbf{d}_i \}$
Skinning weights: $\{ \alpha_i(\mathbf{x}) \}$

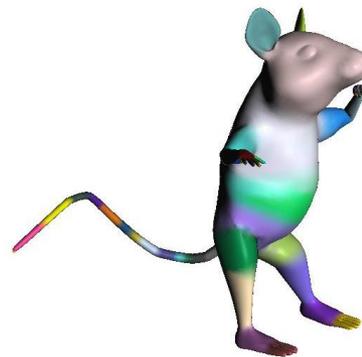
different values estimate each example

Skining equation.

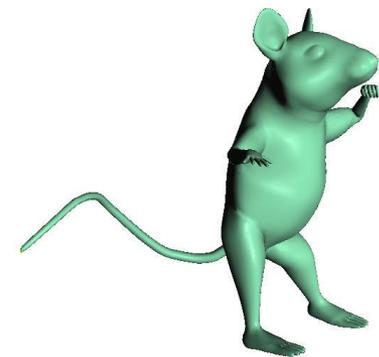
$$\mathbf{v} = \sum_i \alpha_i(\mathbf{x}) \cdot (\mathbf{F}_i \mathbf{x} + \mathbf{d}_i)$$

huge number

20 to 50



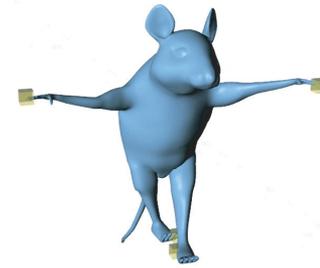
Approx



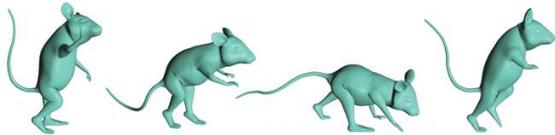
Exact



Inverse Kinematics



- find a semantically proper shape
- obtain natural poses by blending examples



- blend in which domain?
 - vertex positions $\{v_i\}$?
 - control parameters $\{F_i, d_i\}$?

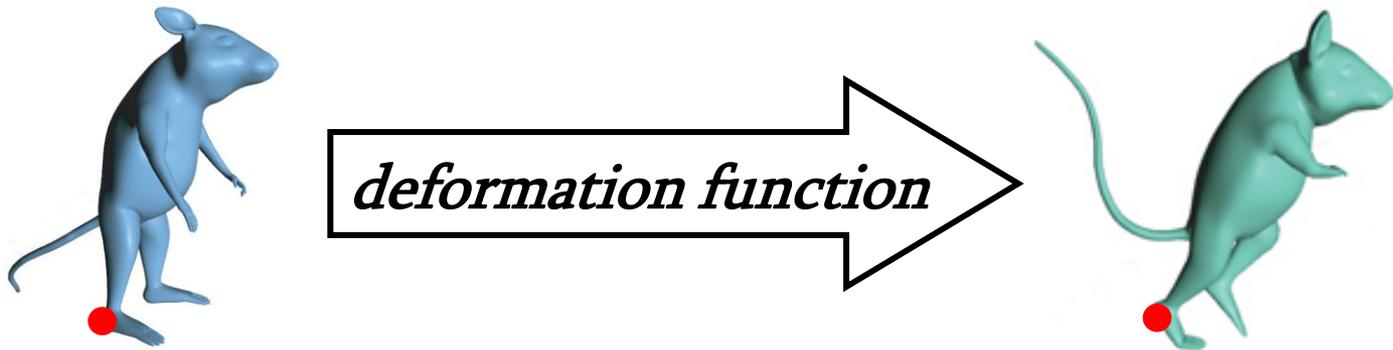
**Blending
function.**

$$m(\beta, f_1, f_2, \dots)$$



Deformation Gradients

- simple 3x3 transformation matrices
- describe deformation of each example



**Skinning
equation.**

$$\mathbf{v} = \sum_i \alpha_i(\mathbf{x}) \cdot (\mathbf{F}_i \mathbf{x} + \mathbf{d}_i)$$

**Vertex
deformation
gradient.**

$$\mathbf{D}_x \mathbf{v} = \sum_i (\mathbf{F}_i \cdot \mathbf{G}_1 + \mathbf{d}_i \cdot \mathbf{G}_2)$$



Deformation Gradients

$$\mathbf{D}_x \mathbf{v} = \sum_i (\mathbf{F}_i \cdot \mathbf{G}_1 + \mathbf{d}_1 \cdot \mathbf{G}_2)$$

$$\mathbf{f} = \mathbf{G} \mathbf{t}$$

flattened control matrices

flattened deformation gradients of vertices

constant matrix

for each example: have $\mathbf{t} \rightarrow$ get \mathbf{f}

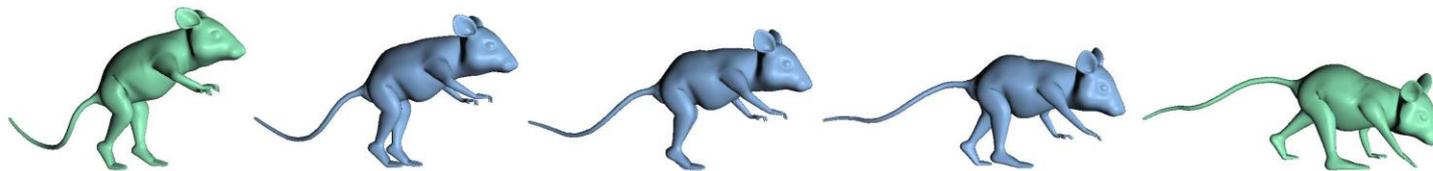


Shape Blending

- combine example deformation gradients

**Blending
function.**

$$m(\beta, f_1, f_2, \dots)$$



- recover the control parameters given deformation gradients

$$t^* = \arg \min_t \|Gt - m(\beta)\|^2$$

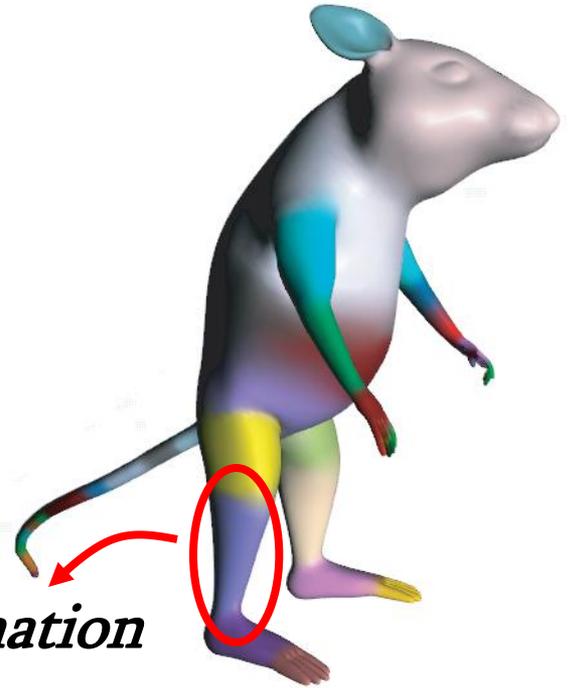
**Has closed form
solution.**

*solving for reduced basis!
but still slow*

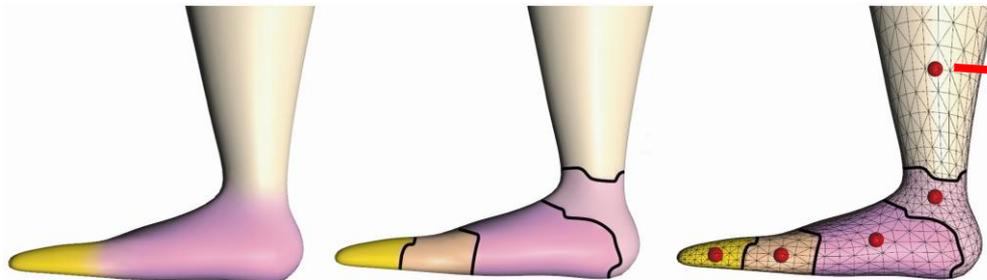


Proxy Vertices

- evaluating deformation gradients at every vertex is undesirable
- summarize using a few vertices



same deformation gradient!



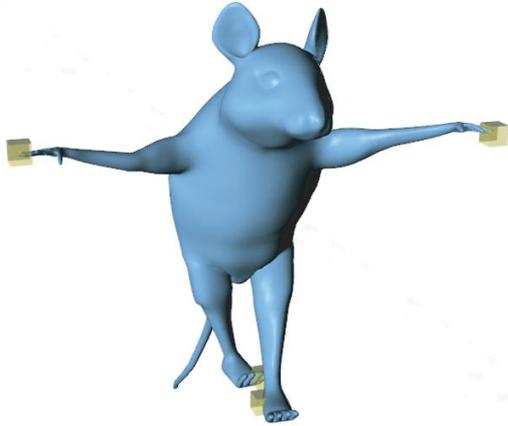
weighted centroid of the group's vertices

controls

vertex groups



Inverse Kinematics



$$\mathbf{t}^*, \beta^* = \arg \min_{\mathbf{t}, \beta} \|\mathbf{G}\mathbf{t} - \mathbf{m}(\beta)\|^2$$

+ constraints

“Find best natural pose that meets user constraints”

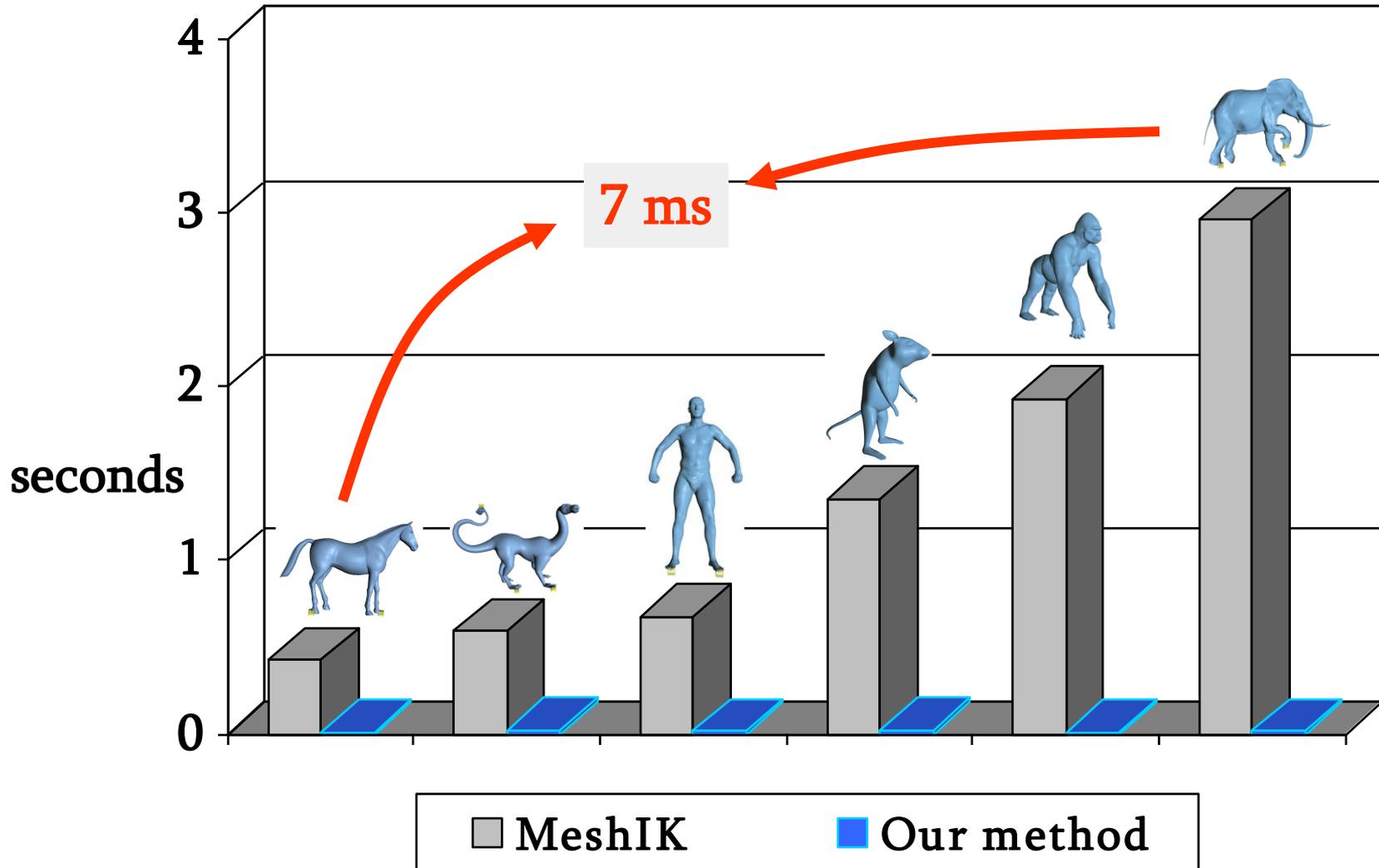
Solving the constrained, nonlinear optimization:

- eliminate constraints
- linearize and iterate
- exploit constants



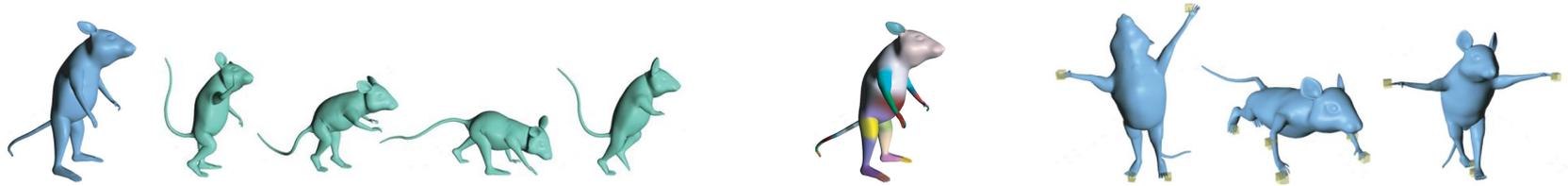
Results

Solving time for interaction



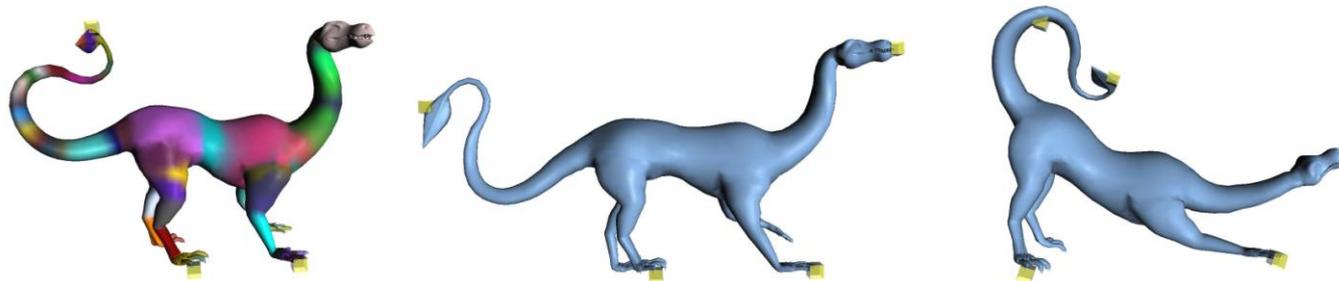
Conclusion

- **interactive control of reduced deformable models**



- **future work**

- error correction for new poses
- model transfer between meshes
- other reduced deformable models



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