Subspace Gradient Domain Mesh Deformation

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Outline

- 1.Introduction
- 2.Methodology
- 3.Results

Introduction

- This paper present a general framework for performing constrained mesh deformation tasks with gradient domain techniques.

Introduction

- The constraints introduce include the ***nonlinear volume constraint*** for volume preservation, the ***nonlinear skeleton constraint*** for maintaining the rigidity of limb segments of articulated figures, and the ***projection constraint*** for easy manipulation of the mesh without having to frequently switch between multiple viewpoints.

Introduction

- To handle nonlinear constraints, ***we cast mesh deformation as a nonlinear energy minimization problem*** and solve problem using an iterative algorithm.
- The main challenges in solving this nonlinear problem are the slow convergence and numerical instability of the iterative solver.

Introduction

- To address these issues,we develop a subspace technique that builds a ***coarse control mesh*** around the original mesh ***and projects the deformation energy and constraints onto the control mesh vertices using the mean value interpolation.***
- The energy minimization is then carried out in the subspace formed by the control mesh vertices. Running in this subspace, ***our energy minimization solver*** is both ***fast and stable*** and it provides interactive responses.

Introduction

- An additional advantage of our subspace technique is that ***it can easily handle real-world mesh output by commercial modelers,*** including meshes having non-manifold features and disconnected components. Such meshes are usually troublesome for existing gradient-domain techniques as they require a “clean” manifold mesh.

Methodology_Overview

- **Deformation with Nonlinear Constraints**
- we can formulate mesh deformation as solving the following unconstrained energy minimization problem

$$\text{minimize} \quad \frac{1}{2}\sum_{i=1}^{m} ||f_i(X)||^2,$$

- where $f_1(X) = LX - \hat{}\delta(X)$
- For convenience we regard $LX = \hat{}\delta(X)$ as a constraint as well and call it the ***Laplacian constraint***.

Methodology_Overview

- Set constraints into two classes, *soft* and *hard constraints*.
- Soft constraint is included as a term in the deformation energy, hard constraint is handled using Lagrange multipliers [Madsen et al. 2004].
- With the hard constraints our energy minimization becomes a constrained nonlinear least squares problem,
- In order to ensure that this nonlinear problem can be efficiently and robustly solved, we need to carefully select soft constraints and reduce the number of hard constraints.

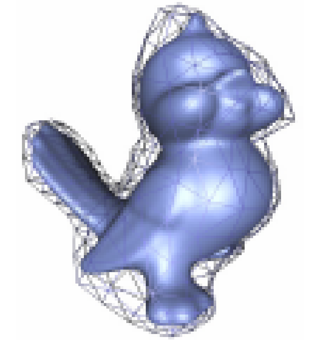
Methodology_Overview

- Allow a nonlinear constraint to be a soft constraint only if it is quasi-linear.
- It can be written as AX = b(X), where A is a constant matrix and b(X) is a vector function whose Jacobian is “very small”
- The Laplacian and skeleton constraints are examples of quasi-linear constraints. Since all nonlinear constraints in the energy function are quasi-linear, energy minimization problem can be written as

$$\text{minimize} \quad ||LX - b(X)||^2 \quad \text{subject to} \quad g(X) = 0,$$

- where L is a constant matrix and g(X) = 0 represents all hard constraints.

Methodology_Overview



- **Subspace Deformation**
- Solving Equation with iterative methods we run into serious problems with slow convergence and numerical instability.
- The subspace method first builds a ***coarse control mesh*** around the original mesh .
- The deformation energy and the hard constraints are then projected onto the control mesh vertices using mean value interpolation .
- Let the control mesh vertices ***P be related to original mesh vertices X through X = WP.*** After projection we perform energy minimization in the control mesh subspace as follows:

$$\begin{array}{ll} \text{minimize} & \|(LW)P - b(WP)\|^2 \\ \text{subject to} & g(WP) = 0. \end{array}$$

Methodology_Detail

- **Skeleton Constraint**
- The user simply specifies a virtual skeleton segment ab

$$\begin{cases} (\mathbf{s}_i - \mathbf{s}_{i-1}) - (\mathbf{b} - \mathbf{a})/r = 0 \\ \quad i = 1, 2, \ldots, r, \\ ||\mathbf{b} - \mathbf{a}|| = \hat{\rho}. \end{cases}$$

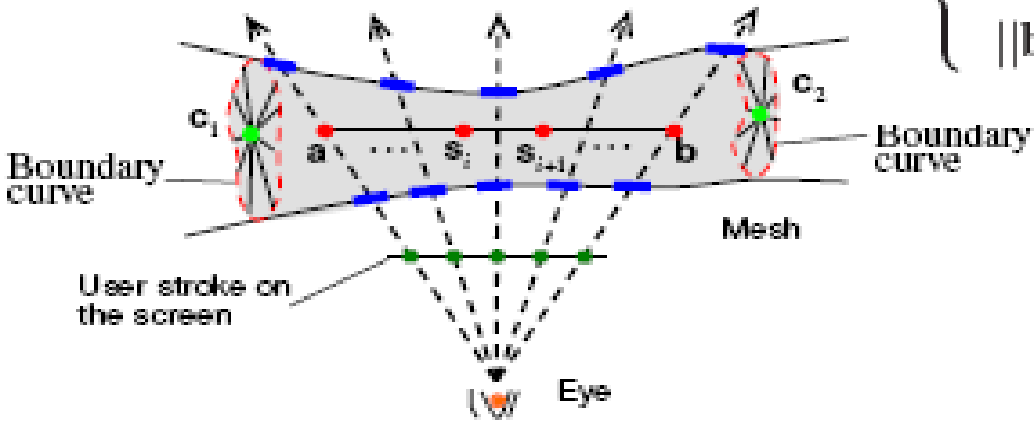


Figure 4: *Skeleton constraint specification. Line segment* $\overline{\mathbf{ab}}$*: constraint bone segment. Dark-green squares: pixels under the user stroke. Blue segments: ray intersections with the mesh. Light-green dots: virtual vertices to close the two open mesh boundaries.*

Methodology_Detail

- We represent each sample point (including a and b) as a linear combination of the mesh vertices:

$$\mathbf{s}_i = \sum_j k_{ij} \mathbf{x}_j$$

- We get

$$\begin{cases} \Gamma X & = & 0 \\ ||\Theta X|| & = & \hat{\rho} \end{cases}$$

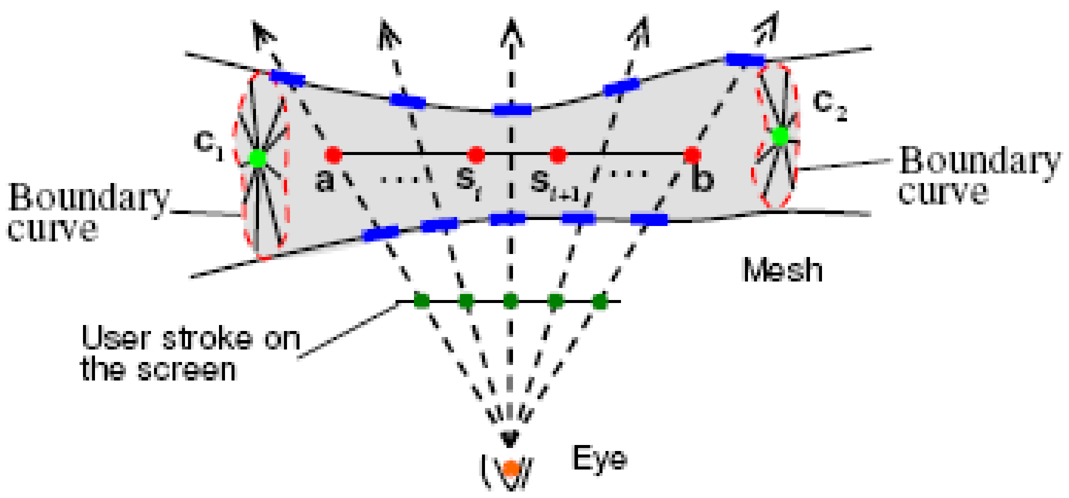
where Γ is a constant $r \times n$ matrix with $(\Gamma)_{ij} = (k_{ij} - k_{i-1,j}) - \frac{1}{r}(k_{rj} - k_{0j})$, and Θ is a row vector with $(\Theta)_j = k_{rj} - k_{0j}$.

Methodology_Detail

- The coefficients kij are computed as the mean value coordinates[Ju et al. 2005] with respect to the constrained part of the mesh.
- Since [Ju et al. 2005] requires a closed mesh, we close the two open ends of the constrained segment by adding as two virtual vertices (c1 and c2 in Figure 4) the centroids of the boundary curves of the open ends.

Methodology_Detail

- **Skeleton Specification**
- user simply draws a stroke over the target region (dark-green) and our algorithm will automatically construct the skeleton segment and the associated constrained region(gray)



Methodology_Detail

- **Volume Constraint**
- The total signed volume of a mesh can be computed using their vertex positions:

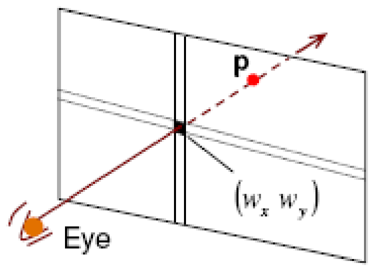
$$\psi(X) = \frac{1}{6}\sum_{T_{ijk}} (\mathbf{x}_i \otimes \mathbf{x}_j) \cdot \mathbf{x}_k$$

- where each Tijk $\in$ K is a triangle formed by vertices i, j, and k. Judging by this, our volume constraint can be easily represented by

$$\psi(X) = \hat{v}$$

Methodology_Detail

- **Projection Constraint**
- The projection constraint is similar to the position constraint for the purpose of user mani



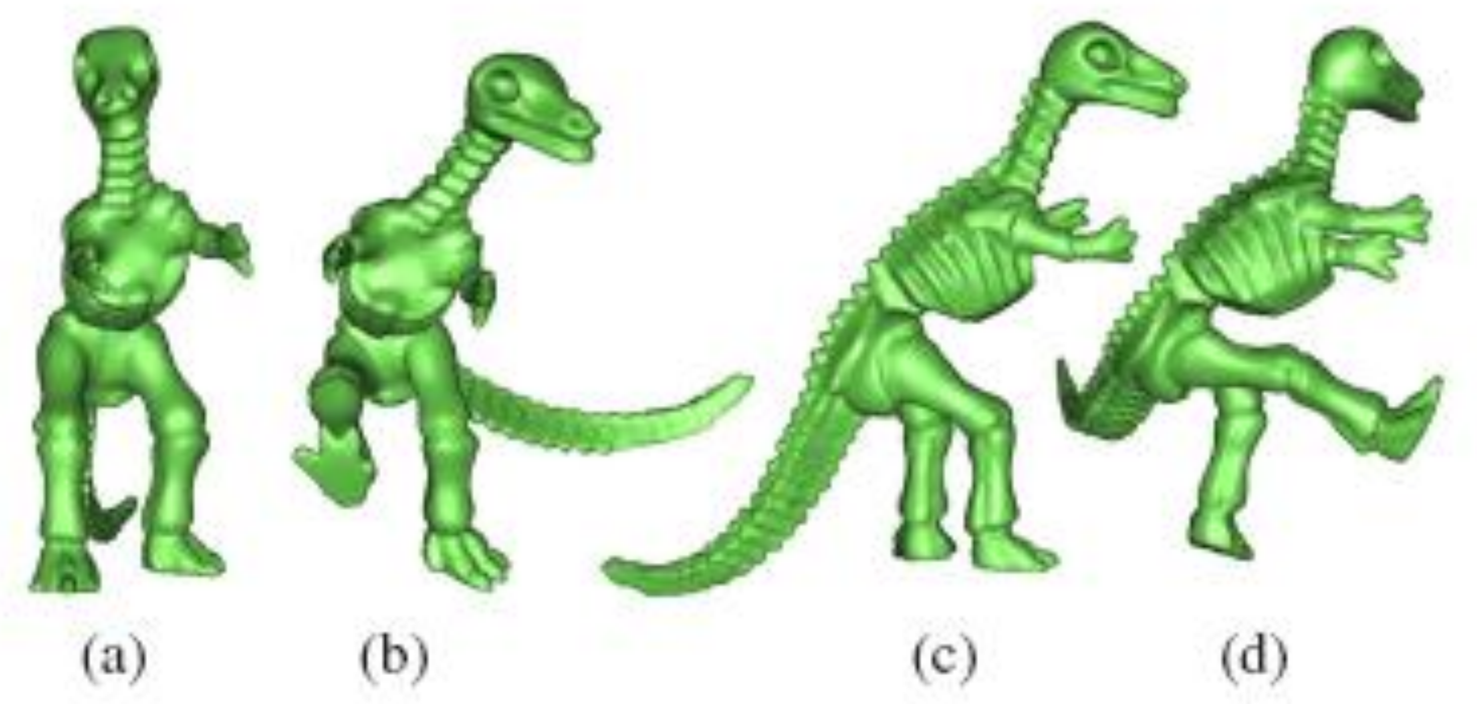
- Let $p = Q_p X$,written as a linear combination of mesh vertex positions X via a constant matrix Q_p
- LetM be themodel viewmatrix whichmaps a point from the object space into the eye space,

Methodology_Detail

$$\left(f\frac{M_x^r \mathbf{p} + M_x^t}{M_z^r \mathbf{p} + M_z^t},\ f\frac{M_y^r \mathbf{p} + M_y^t}{M_z^r \mathbf{p} + M_z^t} \right) = (w_x, w_y)$$

- -> $$\begin{aligned}(fM_x^r - w_x M_z^r)\, Q_p X &= -fM_x^t + w_x M_z^t, \\ \left(fM_y^r - w_y M_z^r\right) Q_p X &= -fM_y^t + w_y M_z^t.\end{aligned}$$
- -> $$\Omega X = \hat{\omega},$$
- where Ω is a constant $2 \times 3n$ matrix and ˆ w is a constant column vector.

Methodology_Detail



Methodology_Detail

$$\text{minimize} \quad ||LX - b(X)||^2 \quad \text{subject to} \quad g(X) = 0,$$

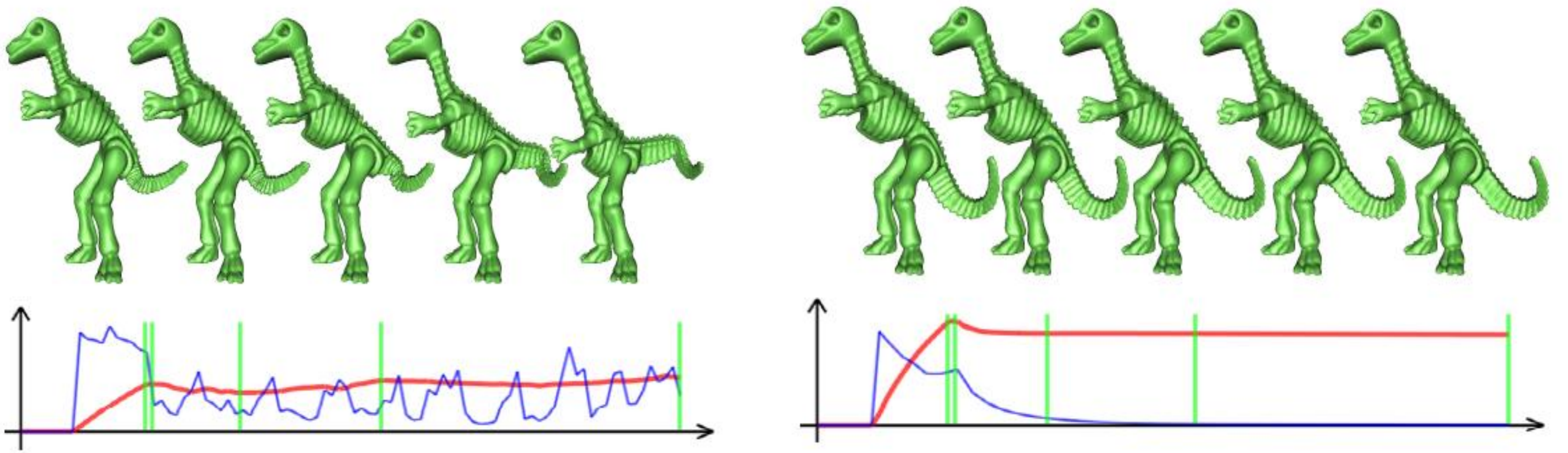
$$L = \begin{pmatrix} \mathcal{L} \\ \Phi \\ \Gamma \\ \Theta \end{pmatrix}, \; b(X) = \begin{pmatrix} \hat{\delta}(X) \\ \hat{V} \\ 0 \\ \hat{\rho}\frac{\Theta X}{||\Theta X||} \end{pmatrix} \text{ and } g(X) = \begin{pmatrix} \Omega X - \hat{\omega} \\ \psi(X) - \hat{v} \end{pmatrix},$$

- where $\Phi X = \hat{V}$ indicates the position constraint

Methodology_Detail

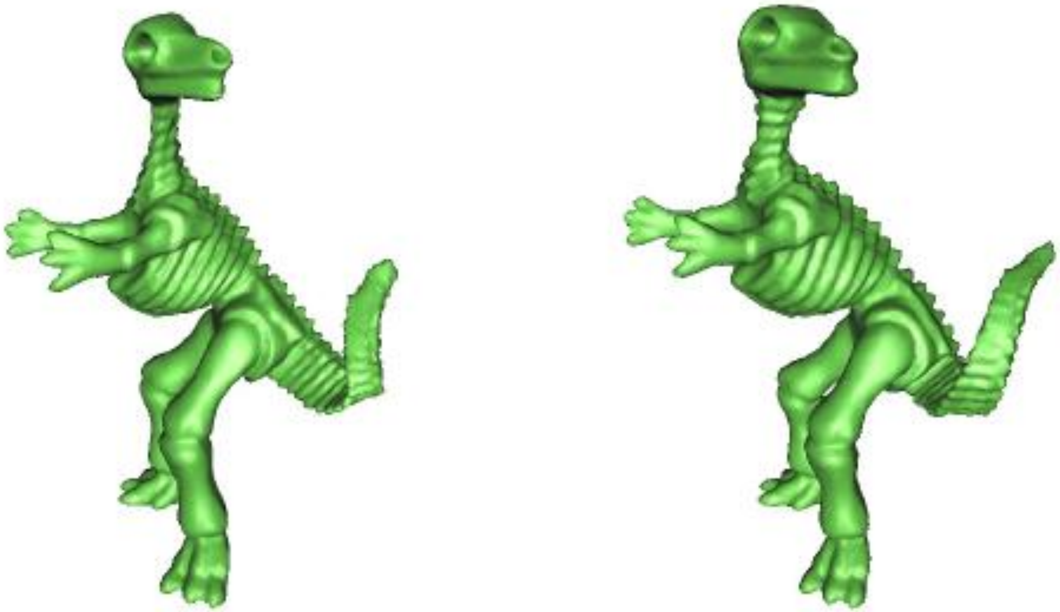
- **Subspace Deformation Solver**
- **The Gauss-Newton Formulation**
- **Numerical Considerations**
- **Convergence and Stability**
- **Subspace Deformation**

Methodology_Detail



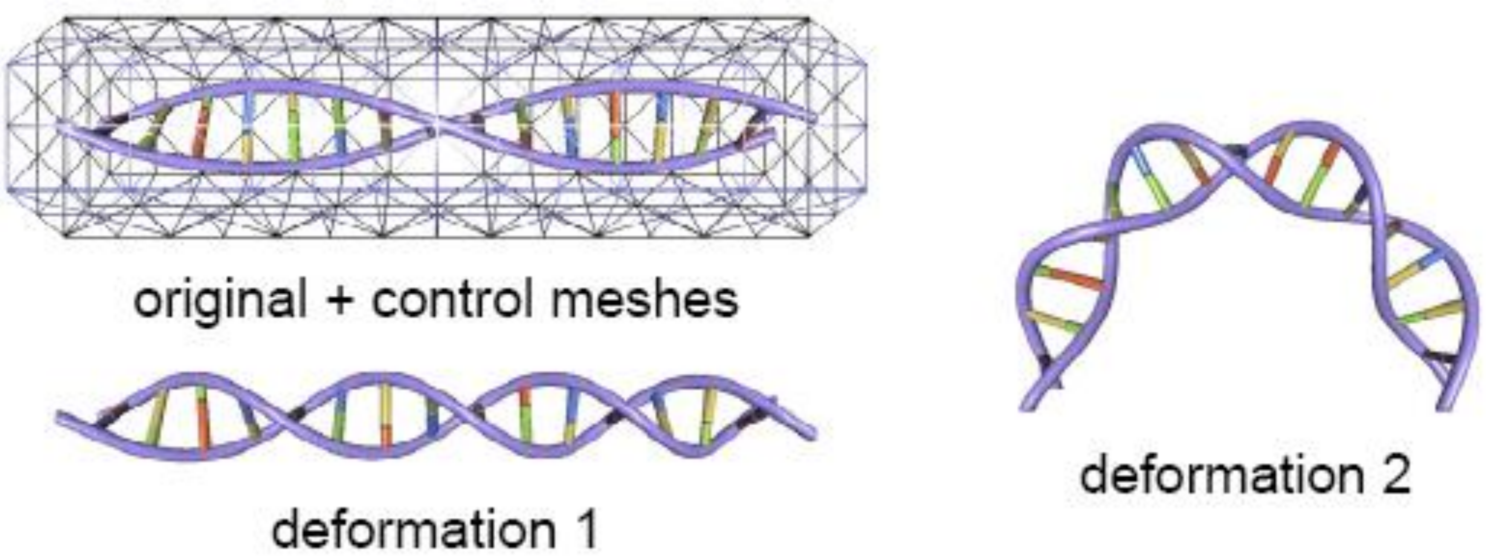
- show an example comparing the stabilities of a direct solver and our subspace solver. As we can see, the subspace solver converges much faster than the direct solver.

Methodology_Detail



- demonstrates a complex example for preserving both volume and surface details; note that our subspace technique generates superior deformation results than naive interpolation.

Methodology_Detail



- using a control mesh in the subspace solver is that it allows us to easily handle non-manifold surfaces or objects with multiple disjoint components.

Results

- Video…