Physics Motivated Modeling of Volcanic Clouds as a Two Fluids Model

Ryoichi Mizuno* Yoshinori Dobashi† Bing-Yu Chen* ‡ Tomoyuki Nishita*

*The University of Tokyo
{mizuno, robin, nis}@nis-lab.is.s.u-tokyo.ac.jp

† Hokkaido University
doba@nis-ei.eng.hokudai.ac.jp

Abstract

In this paper, we present a physics motivated modeling method for volcanic clouds as a two fluids model. Some previous methods model smoke or clouds as one fluid, but the volcanic clouds can not be treated as one fluid. The volcanic clouds consist of the pyroclasts, the volcanic gas and the entrained air. Since the pyroclasts and the volcanic gas can be treated as one fluid, called magma, the volcanic clouds are regarded as two fluids, the magma and the entrained air. The modeling in the 3D analysis space can be simplified to enhance the performance. Since our approach is physics motivated, it can be used to generate physically reasonable and realistic images of volcanic clouds from the volcanic initial eruption to the equilibrium situation.

1. Introduction

Models of volcanic clouds can be used for natural disaster simulations, entertainment, etc. However, there is not many research on the modeling of volcanic clouds. Although some models of volcanic clouds have been proposed, almost all of them are two or less dimensional [5, 7, 9]. Thus, they are not suitable to represent photo-realistic volcanic clouds. A few 3D models have also been proposed recently, but they are very time consuming or not quantitative, i.e. they are only qualitative [6].

This paper presents a modeling method of volcanic clouds based on physical laws that overcomes the above difficulties of previous models. This method is 3D, efficient and quantitatively reasonable. For the quantitatively reasonable simulation, we introduce a model that can treat two fluids and can simulate the mixing of the fluids. We call this model as the "two fluids model (2FM)". In this paper, the mixture of pyroclasts and volcanic gas is called the "magma", which can be regarded as one fluid. The pyroclasts are the rock fragments thrown from a volcano. Although the magma is the molten rock beneath the surface of the earth in geneal, the contents are the same as the magma that we defined. The mixing of the magma and the entrained air that is the prime dynamics of volcanic clouds is simulated as the 2FM. Furthermore, we simplify the model in a physically reasonable manner to make the behaviour of the volcanic clouds more efficient in a 3D analysis space.

‡ At National Taiwan University since August 2003.

2. Related Work

There has been some research on the modeling of volcanic clouds. Woods analyzed the dynamics of volcanic clouds and proposed a numerical model [9]. This model describes only the vertical direction behaviour of volcanic clouds, so that it is a one dimensional model. Thus, it cannot be applied for the representation of the shape of volcanic clouds. Ishimine and Koyaguchi proposed an axisymmetric two dimensional model of volcanic clouds [5]. Although their work visualized the volcanic clouds, the axisymmetric two dimensional model is insufficient for a photo-realistic representation, and is time consuming. Herzog et al. provided 3D simulation results of volcanic clouds [4]. Howver, there is no detailed description of the method of the simulation in [4]. Yngve et al. proposed a method for animating explosions, and represented realistic explosions by solving the governing equations of compressible fluid [10]. However, their method costs a lot of time. Mizuno et al. proposed a 3D model [6] to simulate the behaviour of volcanic clouds efficiently. However, since they used a kind of cellular automaton for the simulation, the model is not quantitatively reasonable but only qualitative. To overcome the difficulties of these previous models, we present a physically-based model to consider the dynamics of volcanic clouds and introduce a physically reasonable simplification of it.

Besides the modeling of volcanic clouds, some useful modeling methods for fluids have also been proposed. Stam [8] introduced the semi-Lagrangian advection scheme to solve the Navier-Stokes equations, which describe the behaviour of fluid. When the time step is large, the calculation of the Navier-Stokes equations is generally unstable.

However, by using the semi-Lagrangian advection scheme, stable simulation of the behaviour of fluid is realized even if the time step is large. Fedkiw et al. provided a technique called the vorticity confinement which is applied to Stam's model [3]. The vorticity confinement can represent small-scale vortices lost during the numerical calculation process. The numerical calculation process we use is based on this method. These methods can be used to simulate smoke or clouds as one fluid, but the volcanic clouds are actually a kind of two fluids model. We therefore introduce the 2FM to simulate the volcanic clouds.

3. Primary Dynamics

The primary dynamics of the volcanic clouds is the mixing phenomenon of the pyroclasts, the volcanic gas and the entrained air. The mixture of the pyroclasts and the volcanic gas is called the magma. Since the pyroclasts are disintegrated due to the momentum of the eruption, the pyroclasts and the volcanic gas reach thermal equilibrium and the relative velocity between them is negligible. Therefore, the magma can be defined as one fluid.

The volcanic clouds is erupted from the vent as a turbulent flow. Just after the eruption, the density of the volcanic clouds ρ is generally several times of that of the surrounding air. Due to the gravity, the velocity of the volcanic clouds, which consist almost completely of magma, slows down rapidly. At the same time, the erupted volcanic clouds entrains the surrounding air; the mixing phenomenon of the magma and the entrained air is called "entrainment". The entrained air, the air in the volcanic clouds, is heated instantaneously by the heat of the magma, and the volcanic clouds therefore expand [7]. The density of the volcanic clouds ρ decreases promptly due to the expansion and becomes less than the atmospheric density ρ_{atm}. Consequently, buoyancy occurs, and the volcanic clouds is pushed upward. Since ρ_{atm} decreases with respect to height, ρ is eventually equal to ρ_{atm} at the upper atmosphere. Then the volcanic clouds loses its upward momentum and spreads horizontally. In this paper, the region where $\rho = \rho_{atm}$ is called the "neutral height".

By using the above phenomena, a typical shape of volcanic clouds with an explosive eruption is illustrated in Figure 1.

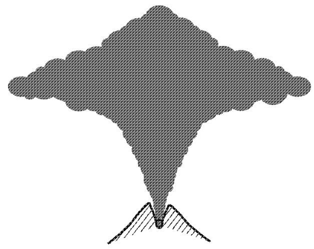


Figure 1. A typical shape of volcanic clouds with an explosive eruption.

4. Modeling

4.1. Evolution of velocity field

The viscosity of the magma is due to the intermolecular attractive force of the pyroclasts, and the viscosity of the entrained air is negligible. Moreover, the eruption velocity of the magma is less than the speed of sound, and the atmospheric fluid can be regarded as incompressible fluid in this case. Hence, in our method, the time evolution of the velocity field is defined by the following non-viscosity Navier-Stokes equations:

$$\nabla \cdot \mathbf{u} = 0, \ \ \frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \nabla p + \mathbf{f}, \tag{1}$$

where $\mathbf{u}$ is the velocity vector, p is the pressure, and $\mathbf{f}$ is the external force vector.

4.2. Evolution of magma and entrained air

The magma and the entrained air are conveyed by the velocity field. Therefore, the following equations can be defined for the time evolutions of the mass of the magma m_m and that of the entrained air m_a:

$$\frac{\partial m_m}{\partial t} = -\left(\mathbf{u} \cdot \nabla\right) m_m, \ \ \frac{\partial m_a}{\partial t} = -\left(\mathbf{u} \cdot \nabla\right) m_a. \tag{2}$$

4.3. Density of volcanic clouds

By adding the volumes of the solid part and gas part per unit mass of the volcanic clouds, the volume of volcanic clouds per unit mass $1/\rho$ is given by

$$\frac{1}{\rho} = \frac{(1-\alpha)(1-n_a)}{\rho_{solid}} + \frac{\{\alpha(1-n_a) + n_a\}RT}{p_{gas}}, \tag{3}$$

where α is the mass fraction of the volcanic gas in the magma, n_a is that of the entrained air in the volcanic clouds, ρ_{solid} is the density of the solid part of the volcanic clouds, R and p_{gas} are the gas constant and the pressure of the gas part of the volcanic clouds, respectively, and T is the temperature of the volcanic clouds. The solid part of the volcanic clouds consists of the pyroclasts. The gas part of volcanic clouds includes the entrained air and the volcanic gas. Then, n_a is defined as follows: $n_a = m_a/(m_m + m_a)$.

The first term of the right hand of Equation (3), $(1 - \alpha)(1 - n_a)/\rho_{solid}$, is the volume of the solid part of the volcanic clouds per unit mass. Since it is less than 1% in general, this term is negligible. Thus, Equation (3) can be approximated as follows by omitting this term

$$\frac{1}{\rho} = \frac{\{\alpha(1-n_a)+n_a\}R_{gas}T}{p_{gas}}. \tag{4}$$

Then, R_{gas} is given by

$$R_{gas} = \frac{\alpha(1-n_a)R_m + n_a R_a}{\alpha(1-n_a)+n_a}, \tag{5}$$

where R_m and R_a are the gas constants of the volcanic gas $(462J/kg \cdot K)$ and the entrained air $(287J/kg \cdot K)$, respectively [5]. Then, T is given by

$$T = \frac{(1-n_a)C_m T_m + n_a C_a T_a}{(1-n_a)C_m + n_a C_a}, \tag{6}$$

where C_m and C_a are the specific heat at constant pressure of the magma $(1847J/kg \cdot K)$ and the entrained air $(1847J/kg \cdot K)$, respectively, and T_m and T_a are the temperatures of the magma and the entrained air, respectively [5]. Since the pressure of the gas part p_{gas} is almost the same as the pressure of the air in the volcanic clouds p_a, we ragard p_{gas} as p_a, and it is given by the gas equation:

$$p_{gas} \approx p_a = \frac{m_a}{V} R_a T_a, \tag{7}$$

where V is the volume of a voxel. Equation (4) can be transformed by substituting Equations (5), (6), and (7) to

$$\rho = \frac{\frac{m_a}{V} R_a T_a}{\alpha(1-n_a)R_m + n_a R_a} \times \frac{(1-n_a)C_m + n_a C_a}{(1-n_a)C_m T_m + n_a C_a T_a}. \tag{8}$$

Therefore, ρ can be denoted as a function of four state variables as $\rho = \rho(m_m, T_m, m_a, T_a)$. The relationship between ρ and n_a is nonlinear. When T_m and T_a are fixed, ρ becomes a function of only m_m and m_a. In our method, the temperatures of the magma and the entrained air are assumed as constants for the following reasons: (1) the thermal capacity of the magma is large, so T_m does not change rapidly; (2) the large part of the entrained air to the volcanic clouds is the atmosphere near the vent, thus T_a can be also regarded as a constant. By these assumptions, a look-up table of the relationship between $\rho/\frac{m_a}{V}$ and n_a can be prepared by a pre-processing. Therefore, calculation of Equation (8) becomes easy and efficient because of the look-up table and the reduction of the state variables from $\rho = \rho(m_m, T_m, m_a, T_a)$ to $\rho = \rho(m_m, m_a)$. Figure 2 illustrates the nonlinear relationship between the normalized density $\rho/\frac{m_a}{V}$ and n_a when $\alpha = 5\%$, $T_a = 300K$, and $T_m = 1000K$.

4.4. Buoyancy

Buoyancy occurs due to the difference of the density of volcanic clouds ρ and that of the atmosphere ρ_{atm}. Therefore, the equation for the buoyancy $\mathbf{f}_{buoy}$ can be defined as

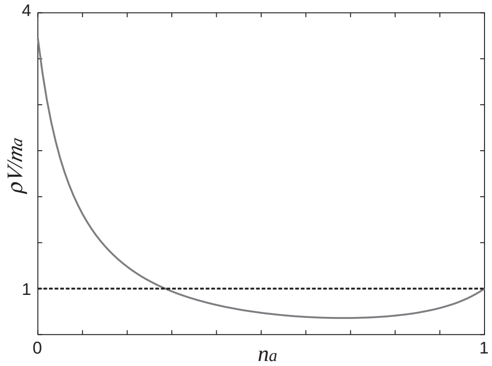


Figure 2. The relationship between $\rho/\frac{m_a}{V}$ and n_a.

follows:

$$\mathbf{f}_{buoy} = g\frac{\rho_{atm}(z)-\rho}{\rho}\mathbf{z}, \tag{9}$$

where g is the gravity constant $(9.8m/s^2)$, $\mathbf{z}$ is a vertically upward unit vector, and ρ_{atm} is the atmospheric density which is defined as the following equation: $\rho_{atm}(z) = \rho_0 \exp(-z/H_e)$, where ρ_0 is the density atmosphere at the ground $(z = 0)$, and H_e is the scale height and is approximately $8km$.

4.5. Numerical calculation

The numerical calculation method we use is based on the fluid solver proposed by Fedkiw et al. [3]. The analysis space is represented as $n_x \times n_y \times n_z$ voxels, where each voxel is a cube with uniform volume V. The velocity vector $\mathbf{u}$, the mass of magma m_m, and that of the entrained air m_a are defined as the state variables at the center of each voxel. As the initial state, $\mathbf{u}$ is set to be a small value by using a random function ($\|\mathbf{u}\| = [0m/s, 1m/s]$), m_m is set to be zero, and $\frac{m_a}{V}$ is set to be the same density of the atmosphere at the corresponding height. However, for the voxels located to the mountain, $\mathbf{u}$ is set to be a zero vector. Then, the eruption velocity $\mathbf{u}_{src}$ and the initial density of magma $\rho_{m,src}$ are assigned to the voxels corresponding to the vent. The time step is chosen to prevent the movement of the erupted magma from exceeding one voxel.

5. Rendering

The volcanic clouds are rendered using the volume data, which represents the density distribution of the magma $\frac{m_m}{V}$. A volume rendering technique is utilized to create realistic images. We assume that the sun is the only light source. As we described previously, the magma consists of large particles such as the pyroclasts and the volcanic gas. This implies that their scattering properties are considered to obey a Mie scattering theory. In this case, it is appropriate to use a Henyey-Greenstein-like function, which was developed by Cornette and Shanks [1], as a phase function. We should take into account multiple scatterings of light when the Mie scattering is dominant. However, the main purpose of this paper is not the realistic image synthesis but the simulation of the volcanic clouds formation. So, we approximate the multiple scattering as a constant ambient term. The light reaching the viewpoint is the sum of the ambient light and the scattered light due to the magma from the sun. These lights are attenuated by the particles before reaching the viewpoint. We utilize a hardware-accelerated rendering method developed by Dobashi et al. [2], originally developed for rendering clouds. For more details see [2].

6. Result

The images generated with our method are shown in Figure 3 when the eruption velocity $\|\mathbf{u}_{src}\| = 100m/s$, $T_a = 300K$, and $\alpha = 5\%$. T_m for Figures 3 (a), (b), (c), and (d) are set to be $700K$, $800K$, $900K$, and $1000K$, respectively. Figure 3 (a) shows round and low volcanic clouds, since T_m is low. Thus, the volcanic clouds can not get enough buoyancy to generate a volcanic column. Figure 3 (b) shows conic volcanic clouds, since the maximum height of volcanic clouds is almost the neutral height. Figure 3 (c) shows mushroom volcanic clouds, since the maximum height of volcanic clouds is beyond the neutral height. The maximum height of volcanic clouds shown in Figure 3 (d) is much greater than the neutral height, i.e., there is overshoot, since T_m is high and the buoyancy generates a lot of upward momentum. Figures 4 (a) and (b) show image sequences of the overshoot and the mushroom, respectively. Since our method is physics motivated, it can simulate volcanic clouds from the volcanic initial eruption to the equilibrium situation. The side wind can also be simulated by controlling the external force in Equation (1). We use the method described in [3] for numerical calculations. To simulate the volcanic clouds, it costs $7sec.$ per time step on average with $150 \times 150 \times 150$ voxels on a desktop PC with an Intel Pentium 4 2.8GHz CPU and 1GB RAM. The volume of each voxel is $100^3 m^3$. To render the simulated results, it costs $2sec.$ per frame on average.

Figure 5 (a) also shows the overshoot phenomenon, and Figure 5 (b) is a real photograph of an overshoot used to compare with it. Figure 6 is the comparison of the result images generated by the proposed simplified model and the original model. Figures 6 (a) and (b) show the conic and the mushroom, respectively. The images generated by the simplified model and the original model are similar. Thus, our simplification is justified. To simulate the volvcanic clouds shown in Figure 6, it costs $10sec.$ per time step on average by using the original model. By using our simplified model, the computation cost for the simulation is $7sec.$ per time step on average.

7. Conclusion and Future Work

In this paper, a physically motivated modeling method for volcanic clouds as a 2FM is presented. Based on the physical laws, the volcanic clouds can be treated as the magma and the entrained air, which is a two fluids model. To enhance the performance, the physically-based model is simplified while keeping the shape of the volcanic clouds un-changed. From our comparison, the shape of volcanic clouds generated by our method looks like that in a real photograph. This is also a proof for the result of our approach.

To simulate the samll and detailed whorls inside the volcanic clouds or on the surface of it is our future work.

Acknowledgments

We thank Nelson L. Max (Univ. of California) and Yujiro Suzuki (The Univ. of Tokyo) for many useful suggestions.

References

[1] W. M. Cornette and J. G. Shanks. Physical reasonable analytic expression for the single scattering phase function. *Applied Optics*, 31(16):3152–3160, 1992.

[2] Y. Dobashi, K. Kaneda, H. Yamashita, T. Okita, and T. Nishita. A simple, efficient method for realistic animation of clouds. *Proc. of ACM SIGGRAPH 2000*, pages 31–37, 2000.

[3] R. Fedkiw, J. Stam, and H. W. Jensen. Visual simulation of smoke. *Proc. of ACM SIGGRAPH 2001*, pages 15–22, 2001.

[4] M. Herzog, H. Graf, and C. Textor. *Three-Dimensional Volcanic Plume Simulations with the Non-hydrostatic Plume Model ATHAM*. Technical Report, Dept. of Atmospheric, Oceanic, and Space Sciences, 2002.

[5] Y. Ishimine and T. Koyaguchi. Numerical study on volcanic eruptions. *Computational Fluid Dynamics Journal*, 8(1):69–75, 1999.

[6] R. Mizuno, Y. Dobashi, and T. Nishita. Volcanic smoke animation using cml. *Proc. of International Computer Symposium 2002*, 2:1375–1382, 2002.

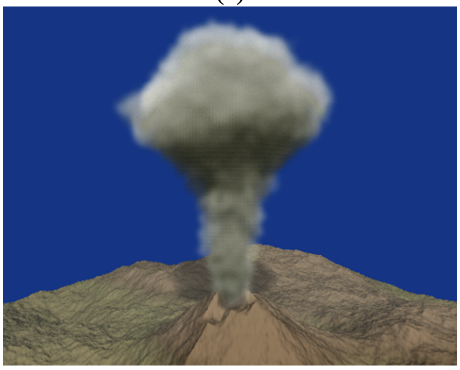
[7] R. S. J. Sparks. The dimensions and dynamics of volcanic eruption columns. *Bulletin of Volcanology*, 48:3–15, 1986.

[8] J. Stam. Stable fluids. *Proc. of ACM SIGGRAPH 99'*, pages 121–128, 1999.

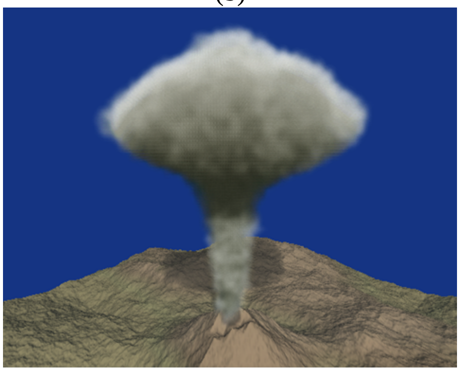
[9] A. W. Woods. The fluid dynamics and thermodynamics of eruption columns. *Bulletin of Volcanology*, 50:169–193, 1988.

[10] G. D. Yngve, J. F. O'Brien, and J. K. Hodgins. Animating explosions. *Proc. of ACM SIGGRAPH 2000*, pages 29–36, 2000.

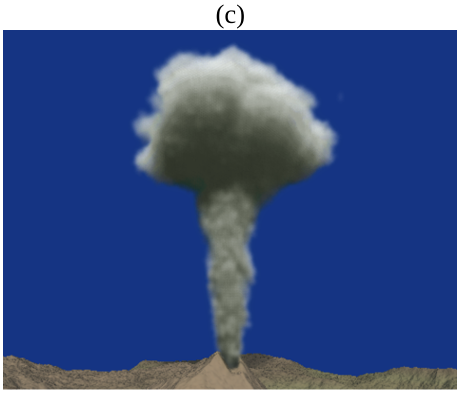
(a)



(b)

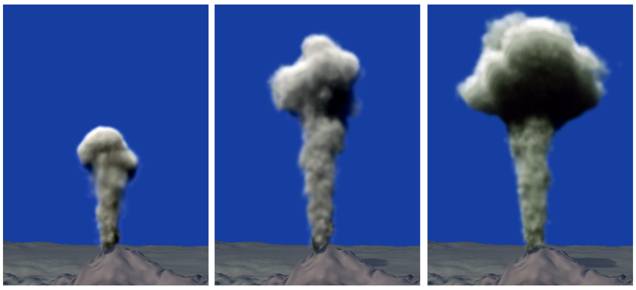


(c)

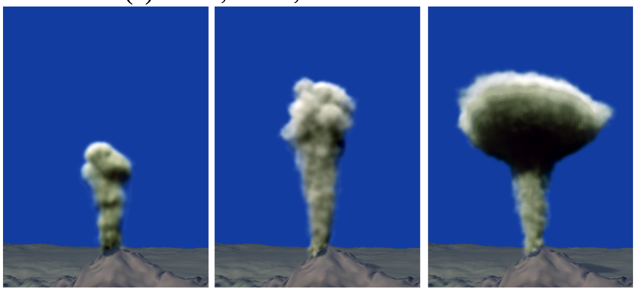


(d)

Figure 3. Different shapes of the volcanic clouds generated with our method by setting different T_m.



(a) 250th, 500th, and 1000th frames.

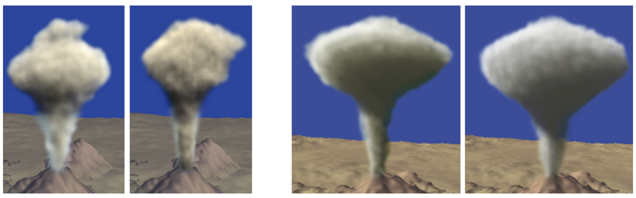


(b) 250th, 500th, and 1000th frames.

Figure 4. Image sequences generated by our method.

(a) (b)

Figure 5. Comparision between (a) a generated image and (b) a photograph.



(a) simplified / original (b) simplified / original

Figure 6. Comparision between the simplified model and the original model.