長さ制約付き糸モデルを用いた破れのシミュレーション
Tearing Simulation using Length-Constrained Yarn Models

1. Introduction

Both cloth simulation and rupture simulation are important areas of research in computer graphics. One of the most common cloth models is based on a simple mass-spring system with some strain limiting method for constraining yarn lengths to achieve inextensibility of cloth. Unfortunately, with strain limiting, the simulation of cloth tearing becomes difficult. In real fabrics, a yarn will break when its strain reaches the breaking point; however, due to the constraint on strain values unrelated to applied forces, actual strain values cannot be directly computed from the simulation result. Hence, we propose a method to support breaking of length constrained yarn by estimating its strain through its stress that would have been exerted if strain limiting were not performed. With our method, various breaking mechanisms of yarns can be performed by only adjusting some parameters.

2. Related Work

Based on elastic model, the fact that most fabrics do not stretch under their own weight leads to the necessity of very stiff springs for a mass-spring cloth model, which induces the numerical instability of simulation [8]. As a result, a variety of different ways for stretch resistance has been continuously proposed; from Provot's iterative post-processing edge constraint [7], to a more recent constraint method based on impulse [2]. Some alternative ways of stabilizing stiff simulation were also proposed; for example, implicit integration method [1], position based dynamics method [6], and fast projection method [4].

These works, and many of their sequels, have a common goal to limit the maximal strain to a threshold value. Accordingly, these kinds of method are suitable for ordinary situation, but they are problematic in the case of excessive stretch, or when rupture should be occurred. The method proposed in [5] describes how to deal with tearing cloth. However, it tears cloth by not exactly enforcing strain limits; when and where yarns are cut were not directly related to applied external forces and cloth material properties, but dependent on a strain limiting algorithm.

3. Proposed Yarn Tearing Simulation Model

Our method adds tear-ability to length-constrained yarn models, while benefiting from their numerical stability. We first describe an ideal yarn model in Sec. 3.1; as in material science, the strength and elongation of a yarn in our work are in reference to its stress-strain curve. To determine when and where to cut yarns, Sec. 3.2 describes a method for estimating strain values, not from length-constrained simulation results themselves, but by calculating tensions that would have acted on yarns if length constrains were not used. Once strain values are obtained, we can reproduce characteristic behaviors of a yarn, where it slightly extends elastically as predicted by Young modulus (Sec. 3.3), and then begins irreversible deformation when stress goes beyond the yield point (Sec. 3.4).

3.1 Ideal Yarn Model

An analysis of the stress-strain curve of a fiber (Figure 1) reveals that the relationship is a fairly steep line in the initial region, which means yarns are almost inextensible until the yield point (see Sec. 3.4) is reached. The ratio of the stress over the strain in the initial region is known as Young modulus.

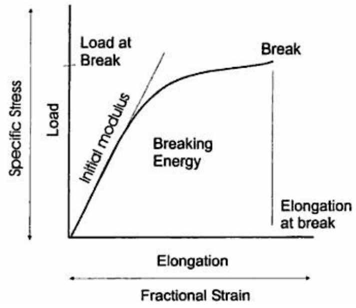

Figure 1: Typical stress-strain curve of a fiber [3]

In ideal yarn simulation based on a mass-spring system, extremely stiff springs are needed to depict large Young modulus of yarns. An example of a mass-spring system is shown in Figure 2. Given a mesh of n vertices and q edges, the total forces acting on a vertex i are the sum of all external forces F_{ext_i}, highly stiff spring forces F_{stiff_i}, and other internal forces F_{in_i}.

$$F_i = F_{ext_i} + F_{stiff_i} + F_{in_i}$$

Owing to a net force, its velocity and position could be advanced from v_i and p_i to $v_i^{'}$ and $p_i^{'}$. However, to avoid the stiff equations, which are numerically unstable, non-stiff springs are frequently used instead, then some length constraints are applied to gain the final velocity $v_i^{''}$ and position $p_i^{''}$. As a result, the strain values are no longer related to the stress-strain curve of a yarn.

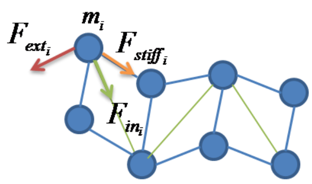

Figure 2: An example of a mass-spring system

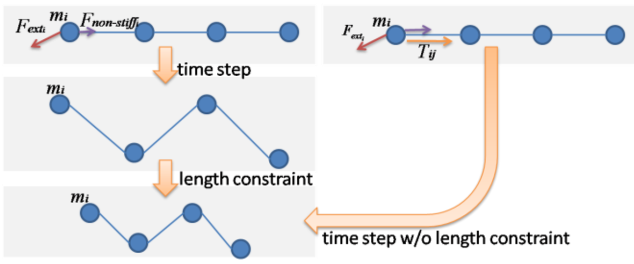

Figure 3: The idea of our system. We perform ordinary length-constrained yarn simulation (left), where we first advance the system by some time step using non-stiff spring, followed by application of length constraints. Our idea is to compute tension T_{ij} of stiff spring forces (right) that would produce equivalent simulation results after time stepping without length constraints

3.2 Tension Computation

Our scheme is to find the tensions T_{ij} of each edge that would have been exerted by stiff springs and would have brought the system to $v_i^{"}$ and $p_i^{"}$ without strain limiting (Figure 3). From tension T_{ij} and the yarn radius r, the stress can be computed as $\sigma_{ij} = T_{ij}/\Pi r^2$, from which the strain for each edge can be obtained using the stress-strain curve. Then, a yarn will be cut when this strain value reaches the strain at breaking point.

Assuming forward Euler integration, if a vertex i with mass m_i connected to k edges is simulated with time step Δt, the following equation must be satisfied in order to match the updated velocity to the length-constrained simulation result $v_i^{"}$.

$$v_i^{"} = v_i + \frac{\Delta t}{m_i}\left(F_{ext_i} + F_{in_i} + \sum_{j=0}^{k} T_{ij}\right)$$

In case of yarn, where the number of edges is less than the number of vertices($q < n$), these systems can be directly solved for T_{ij}. However, in case of cloth ($q > n$), this is an under-determined problem; since q edges require q unknowns, but there are only n equations for n vertices. In this case, we can estimate tension T_{ij} by utilizing least squares solution with regularization that keeps T_{ij} as small as possible.

$$\min_{T_{ij}}\left(\sum_{i=0}^{n}\left\|v_i + \frac{\Delta t}{m_i}\left(F_i + \sum_{j=0}^{k} T_{ij}\right) - v_i^{"}\right\|^2 + \lambda\sum_{i=0}^{n}\sum_{j=0}^{k}\|T_{ij}\|^2\right)$$

Here, λ is a regularization parameter that controls tradeoff between accuracy and the empirical risk. In our test system, lambda is set to 10^{-6}, which is small enough to get a reasonable solution, without system unstable.

3.3 Updating Strain Limit

As can be seen in the stress-strain curve (Figure 1), yarn is lengthened according to stress applied. To include this behavior into the simulation, we perform a threshold updating for strain limitation in accordance with Young modulus. Without external applied forces, the threshold value is set to near zero, while the value is changed according to stresses (obtained as described in Sec. 3.2) when some forces are applied.

3.4 Handling Yield Points

A yield point is the point at which a material begins to deform plastically. Once the yield point is passed, the material will not return to its original shape, even when the applied force is removed. This characteristic is also merged to our model by updating the rest length of an edge when its stress and strain exceed the yield point.

In addition, the failure behavior of a real yarn is changed according to the strain rate. That is, the more time is taken to break a yarn, the less vigorousness of breakage will be. The increment of the rest length not only affects the final length of a yarn after breakage or removal of external forces, but also reduces spring forces of simulation at low strain. Thus, different failure of yarns behaviors at different rates of extension can also emerge.

4. Results

We have conducted several experiments to demonstrate that our method can produce different kinds of yarn breaking behaviors, while maintaining numerical stability thanks to the length constraints. The computation time for simulating yarn having 25 vertices was 0.1 second per frame..

Figure 4 shows a comparison of breaking behaviors of two kinds of yarns as animation sequences. To make the right side yarn stronger than the left side yarn, we set the Young modulus and also breaking point of the right side yarn larger than the left side one. So, when the same amount of force is applied to both yarns, the weaker one will be torn earlier and more vigorously.

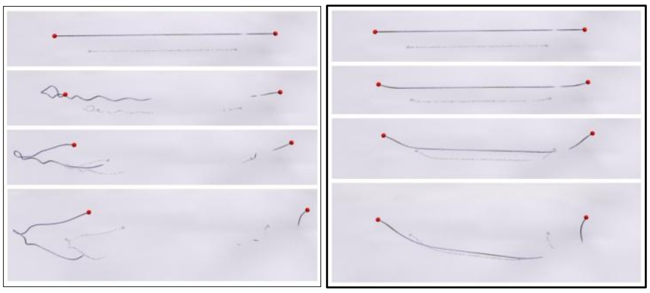

Figure 4: Example frames from animations of 2 kinds of yarns tearing. The animation sequences are ordered from top to bottom. The weaker yarn on the left is actually torn earlier, but here we match the frames for a comparison purpose.

For further detail and some more example results, please see supplementary material.

5. Conclusions and Future Work

We have demonstrated a yarn simulation technique that allows both inextensibility and tear ability. As our scheme can be implemented as an additional step, the integration into an existing simulation system is effortless. A variation in the quality of yarns can be achieved by only changing Young modulus, yield point or breaking point.

Although only yarns are considered in this paper, we believe that the presented approach could be easily extended to handle cloth as well, which we plan for future work.

References

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