Mathematical Analysis of Algorithms

Homework #1
Due Date: 
Reading Assignment: Preface, Chapter 1, 2.1, 2.2
Problems:

1. 1–9
2. 1–16
3. 2–11
4. 2–22
5. Prove or disprove that the Knuth Sequence defined by

\[ K(0) = 1; \]
\[ K(n + 1) = 1 + \min \left( 2K(\lfloor \frac{n}{2} \rfloor), 3K(\lfloor \frac{n}{3} \rfloor) \right), \text{ for } n \geq 0, \]

has the property that \( K(n) \geq n, \) for \( n \geq 0. \)
(The sequence begins 1, 3, 3, 4, 7, 7, 7, 9, 9, 10, 13, \ldots )

6. Consider the series of fractions

\[
\frac{1}{2}, \frac{1/2}{3/4}, \frac{1/3}{5/8}, \frac{1/4}{5/6}, \frac{5/6}{7/8}, \ldots
\]

Suppose that each fraction is simplified to be a fraction of two products of integers (for example, the third is \( \frac{14 \cdot 6}{3 \cdot 5 \cdot 8} \)). Prove that, for the \( n^{\text{th}} \) fraction, the sum of the \( k^{\text{th}} \) powers of the numbers in the numerator equals the sum of the \( k^{\text{th}} \) powers of the numbers in the denominator for \( 0 \leq k < n. \) (For example, \( 1^2 + 4^2 + 6^2 + 7^2 = 2^2 + 3^2 + 5^2 + 8^2. \) )