Mathematical Analysis of Algorithms

Homework #1
Due Date:
Reading Asignment: Preface, Chapter1, 2.1, 2.2
Problems:

1. 1–9

2. 1–16

3. 2–11

4. 2–22

5. Prove or disprove that the *Knuth Sequence* defined by

$$K(0) = 1;$$
$$K(n+1) = 1 + \min\left(2K(\lfloor\frac{n}{2}\rfloor), 3K(\lfloor\frac{n}{3}\rfloor)\right), \quad \text{for } n \geq 0,$$

   has the property that $K(n) \geq n$, for $n \geq 0$.
   (The sequence begins $1, 3, 3, 4, 7, 7, 7, 9, 9, 10, 13, \dots$)

6. Consider the series of fractions

$$\frac{1}{2}, \ \frac{1/2}{3/4}, \ \frac{\frac{1}{2}/\frac{3}{4}}{\frac{5}{6}/\frac{7}{8}}, \ \frac{\frac{1/2}{3/4}/\frac{5/6}{7/8}}{\frac{9/10}{11/12}/\frac{13/14}{15/16}}, \ \dots$$

   Suppose that each fraction is simplified to be a fraction of two products of integers (for example, the third is $\frac{1\cdot4\cdot6\cdot7}{2\cdot3\cdot5\cdot8}$). Prove that, for the n^{th} fraction, the sum of the k^{th} powers of the numbers in the numerator equals the sum of the k^{th} powers of the numbers in the denomerator for $0 \leq k < n$. (For example, $1^2 + 4^2 + 6^2 + 7^2 = 2^2 + 3^2 + 5^2 + 8^2$.)