Mathematical Analysis of Algorithms

Homework #5
Due Date:
Reading Assignment: Chapter 5
Problems:

1. 5–14

2. 5–43

3. 5–58

4. 5–24

5. Evaluate $\dfrac{1}{N}\displaystyle\sum_{1\le k\le N}\sum_{t}\frac{t\binom{N-k}{t}\binom{k-1}{t}}{\binom{N-1}{k-1}}$

6. The **Merge Sort** program sorts n numbers $X(1), X(2), \ldots, X(n)$ by:

1. If $n = 1$, then do nothing. Otherwise, do Steps 2 through 4.
2. Sort the $\lfloor n/2 \rfloor$ numbers $X(1)$, $X(2)$, $\ldots$, $X(\lfloor n/2 \rfloor)$ by calling **Merge Sort** recursively.
3. Sort the $\lceil n/2 \rceil$ numbers $X(\lfloor n/2 \rfloor + 1)$, $X(\lfloor n/2 \rfloor + 2)$, $\ldots$, $X(n)$ by calling **Merge Sort** recursively.
4. Merge the two sorted lists from Steps 2 and 3. (This takes $n - 1$ comparisons in the worst case, using the obvious algorithm.)

Let $S(n)$ be the worst-case number of comparisons needed by **Merge Sort** to sort n numbers. The above description of the program shows that

$$\begin{aligned} S(n) &= S(\lfloor n/2 \rfloor) + S(\lceil n/2 \rceil) + n - 1, \quad n > 1; \\ S(1) &= 0. \end{aligned}$$

Solve the recurrence for the general case, for $n = 1, 2, 3, \ldots$

7. Give the combinatorial interpretations of the following two identities:

1. For $0 \leq m \leq n$, $\displaystyle\sum_{k\geq 0} \binom{n}{k}\binom{k}{m} = \binom{n}{m}2^{n-m}$
2. $\displaystyle\sum_{1\leq k\leq n} H_{m-1}^{k} = H_{m}^{n}$